Public Patents, Private Secrets, Persistent Monopolists and Radical Innovators

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Abstract

Why Coca Cola keeps secret the formula of its famous soft drink? Why pharmaceutical companies with a lot of patents keep innovating? Why sports based on advanced technologies as Formula 1 racing are characterized by periods of persistence of leadership and eventual leapfrogging? Is it true that outsider firms invest in more radical innovations? I provide some answers to questions like these building on models of innovation. I emphasize the trade-offs existing between patenting and not and the consequences of this choice on the R&D activity. Then I develop models of innovation with sequential entry which rationalize innovation by patentholders and partial persistence of technological leadership. Finally I show that in such a model patentholders look for more radical innovations than the entrants.

1 Introduction

This paper provides some answers to some real world questions like: why Coca Cola keeps secret the formula of its famous soft drink? Why pharmaceutical companies with a lot of patents keep innovating? Why sports based on advanced technologies as Formula 1 racing or the America's Cup are characterized by periods of persistence of leadership and eventual leapfrogging? The models I develop will provide different endogenous characterizations for the value of innovations. The results have wide applicability in markets where innovations

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are important and especially in markets for new technologies which are the engine of technological progress, hence I will emphasize the consequences of my results for the theory of growth based on innovations.

First, I emphasize the trade-offs existing between patenting and not and the consequences of this choice on the R&D activity. When a firm discovers a new technology, there are two options: patenting it and obtaining sure profits for a period of time or keeping it secret and obtaining profits as long as noone else finds out a similar or better technology and patents it. Understanding what may induce a company to choose the first or the second option is important especially when designing a patent law.

Second, I discuss models of innovation with sequential entry which rationalize innovation by patentholders and partial persistence of technological leadership. I will study a patent race where the patentholder has the opportunity to make a strategic precommitment to a level of investment in R&D. This may happen through a specific investment in laboratories and related equipment for R&D or by hiring researchers. I first consider a single patent race where each participant hires a number of researchers until the new technology is discovered. For a given number of firms, a patent race based on Stackelberg competition delivers an equilibrium in which the patent holder invests less than each other entrant. Indeed, not only is this result qualitatively similar to the one under Cournot competition, but also stronger, since I prove that the leader always invests less than in the Cournot case. Moreover each of the outsiders invests less than in the Cournot equilibrium, so that the overall R&D activity is lower. Under free entry, the results are completely changed and I derive a crucial application of a more general result by Etro (2002, 2004) on Stackelberg competition with free entry: the leader patentholder always invests in R&D and more so than any other firm, thus the Stackelberg assumption with free entry delivers a new rationale for the investment by incumbent monopolists in R&D and for the persistence of a monopoly. I study some examples and investigate how the market structure is affected by the If we believe that Stackelberg competition is the right assumption in the study of patent races, we obtain very strong conclusions from this analysis: a market characterized by some persistence of monopoly is competitive, while one with continuous leapfrogging must be characterized by some barriers to entry!

Finally I employ the same model to investigate who between the patentholder and the entrants has more incentives to develop more radical innovations. In general, Stackelberg competition with free entry and an endogenous amount of innovation implies a higher investment for innovations of greater value for the leader patentholder than for each of the other firms. This result is driven by the complementarities between the investments in more radical and more rapid innovations and overturns the usual hypothesis that outsiders make more radical innovations. As a matter of fact, empirical evidence by Blundell, Griffith and Van Reenen (1999) is consistent with our result: they show that, after an innovation, the market value of firms increases more for firms with stronger market power, which is consistent with the argument that "leading firms have a systematic tendency to produce innovations that are intrinsically of higher quality than smaller firms".

The paper is organized as follows. Section 2 of the paper describes the choice between patenting and not. Section 3 studies patent races with Cournot competition and compares them with those under Stackelberg competition. Section 4 studies the endogenous size of innovations and section 5 concludes.

2 Public Patents versus Private Secrets

In a safe deposit vault at the Trust Company of Georgia, USA, lies the secret of one of the worlds most popular soft drinks, Coca Cola.[1] And, it is said, only the company directors can authorise the opening of the vault. Although numerous outlets around the world have a franchise to bottle or can and distribute Coke, none knows the precise ingredients. They are simply supplied with syrups and other ingredients from the Coca Cola company- and mix them either carbonated water. Many competitors[2] and even the USA government[3] have tried to discover the secret formula of Coca Colas distinctive flavour. But none has yet succeeded.

[1] It was May of 1886 in Atlanta when the pharmacist Dr. Pemberton's concocted caramel-coloured syrup in a three-legged brass kettle in his backyard. He first "distributed" the new product by carrying Coca-Cola in a jug down the street to Jacobs Pharmacy. For five cents, consumers could enjoy a this syrup added of carbonated water. The bookkeeper of the pharmacy, Frank M. Robinson, suggested the name and penned Coca-Cola in the unique flowing script that is famous worldwide today. As the company expanded, the new owner Candler could not prepare the syrup all himself, so the ingredients were all simply labelled 1 to 9 and the managers at the branch factories were only told the proportions required and the mixing procedure. By the turn of the century Candler would become one of the wealthiest men in Atlanta and Coca Cola would become the most popular soft drink in America.

[2] What is the taste difference between Coca-Cola and Pepsi? Standard colas are usually flavoured with an orange-lemon-lime behind the vanilla, coca and kola tastes. Coke is more orange-biased while Pepsi is more lemon-flavoured. Also, the sugar and carbonation is different, with Pepsi being sweeter and a little flatter.

[3] In 1909 the US federal government impounded 40 barrels and 20 kegs of Coca Cola and charged the company with violation of the Pure Food Act, because the Coca ingredient implied the presence of cocaine. But during the trial various counter appeals continued for nearly ten years none of the evidence given could find traces of cocaine in the form of the coca extract, nor cola. Yet a witness for the company which supplied Coca Colas ingredients No 5 described how it was made from Coca leaf, with its cocaine content removed, and extract of cola nut.

However, the American author William Poundstone did some painstaking research when he published in his 1993 book "Big Secrets". He suggests that Coke basics ingredients- number one to nine and known as merchandises by the company as follows: 1) sugar, 2) caramel, 3) caffeine (although there is now a caffeine free version available), 4) phosphoric, 5) coca leaf extract (with its cocaine content removed) and a small amout of cola nut extract, 6) citric acid and sodium citrate, 7) lemon, ornage, lime, cassia (a type of cinnamon), nutmeg oils, and probably others, 8) glycerine, 9) vanilla. Although the proportions of some of these ingredients all mixed with carbonated water can be discovered by chemical analysis; the most important and most elusive is the mixture of essential oils is merchandise 7).

The flavour of the mixture is not simply the sum totals of the oils, because other flavours are created by the interaction of the oils. Anyone trying to reproduce the mixture would need to know the exact ingredients which are difficult to analyse with certainty and their precise proportions, with hitherto defied analysis. *It is rumored that not less than two and no more then three picked people ever know it at the same time, and they never travel together.*

2.1 A simple model of patenting

This section tries to rationalize why certain innovating firms like Coca Cola do not patent their inventions and evaluates the economic consequences of this choices from a positive and normative point of view.

A firm with an innovation faces a dilemma: is it better to patent the discovery or to keep it secret? On one side, patenting provides a certain flow of monopolistic profits, let's say π, for a determinate period of time which is the lenght of the patent, let's say T. On the other side, keeping secret the discovery allows to obtain the same flow of profits until someone else will find out the same technology (or even a better one) and will patent it. Only if the probability of imitation is great enough or the lenght of the patent is long enough, patenting is the prefered choice. The possibility of reverse engeneering would make patenting always the best option. However reverse engeneering is not always possible. In some cases, as the Coca Cola case suggests, it is almost impossible. Clearly, unused patents are inefficient since they avoid diffusion of information and delay further innovation.

More formally, let's denote with r the interest rate and with p the arrival rate of innovations according to a standard Poisson process. Then the value of patenting an innovation is:

$$\mathbf{V}^P = \int_o^T \pi e^{-rt} dt = \frac{\pi \left[1 - e^{-rT}\right]}{r} \tag{1}$$

while the value of hiding the innovation is:

$$\mathbf{V}^H = \frac{\pi}{r + p} \tag{2}$$

It follows that patenting is the choice if and only if $\mathbf{V}^P > \mathbf{V}^H$, that is:

$$p > r \frac{e^{-rT}}{1 - e^{-rT}} \equiv \bar{p}(T, r) \tag{3}$$

where the cut-off $\bar{p}(T, r)$ is decreasing in T - a longer patent makes easier to prefer patenting - and typically decreasing in r - an higher interest rate reduces the value of hiding while it has ambiguous but small effect on the value of patenting.[4] In general if the probability of imitation is high enough patenting will be the best choice.[5]

2.2 Implications

One can study in depth the consequences of such an endogenous patenting choice taking in account further crucial aspects.

1) First, the probability of imitation p is an endogenous variable, because other firms will invest to obtain a patent whose value is endogenous, and it is exactly the value of patenting the same innovation - notice that the imitator will immediately patent the innovation to drive out of the market the initial inventor who was hiding the innovation. If we just assume that $p = p(\mathbf{V}^P)$ with $p' > 0$, the condition for patenting (3) becomes:

$$p\left(\frac{\pi \left[1 - e^{-rT}\right]}{r}\right) > \bar{p}(r, T)$$

[4] *Proof*: use the Taylor series expansion $e^{-rT} \approx 1 - rT$, which is valid for r small, to obtain $V^P \approx \pi T$ and:

$$\bar{p}(T, r) = r \frac{e^{-rT}}{1 - e^{-rT}} \approx r \frac{1 - rT}{rT} = \frac{1}{T} - r$$

from which $\partial \bar{p} / \partial r \approx -1 < 0$ and $\partial \bar{p} / \partial T \approx -T^{-2} < 0$. Q.E.D.

[5] The assumption that the flow of profits is constant over time and that innovations arrive with a Poisson process are crucial to derive a simple choice between patenting and hiding. If we drop them a strategy of the kind “hide until time $\bar{t}$ and then patent” could be optimal.

which tells us that patenting will be the best option if and only if:

$$\pi > \bar{\pi}(T, r) \tag{4}$$

where the cut-off $\pi(T, r)$ is now decreasing in T and in r.[6] Further microfoundation of the function $p(.)$ can be obtained by imagining a patent race between potential imitators.

2) Second, the amount of initial innovation is an endogenous function of the prize, which is now:

$$V = max\left(\mathbf{V}^P, \mathbf{V}^H\right) = \left\{ \begin{array}{ll} \frac{\pi}{r+p(\pi[1-e^{-rT}]/r)} & \text{if} \quad \pi \leq \pi(T, r) \\ \frac{\pi\left[1-e^{-rT}\right]}{r} & \text{if} \quad \pi > \pi(T, r) \end{array} \right\}$$

hence the patenting versus hiding decision affects the amount of equilibrium innovation in a non trivial way. For instance, in the patenting regime an increase in the lenght of the patent T increases V and so the initial investment in R&D, but in the hiding regime, an increase in T just gives incentives to imitation activity, so as to reduce V and consequently the initial investment in R&D. The initial market for innovation can be modeled as a context between potential inventors, but the results are quite different if we are in the hiding regime.

3) Third, in general equilibrium also the profit flow π and the interest rate r are endogenous variables. The flow of profit depends from the size of the market, which derives from aggregate variables and is affected by growth, and the size of innovations. Firms may endogenously decide the size of their innovations: the choice would be between small innovations to keep secret and drastic innovations to patent. Finally, if there is a sequence of available innovations the choice between patenting and hiding as to take in account future patent races.

4) If we consider sequences of innovations as the engine of growth following Aghion and Howitt (1992) we should take in consideration the effects of the patenting versus hiding decision on growth. The patenting regime which, according to (4) holds for big enough innovations, is given by the Aghion-Howitt case extended to patents with finite life. The imitation regime in which new innovations are kept secret and other firms invest to discover and patent what somebody already knows, emerges if innovations are small enough. The latter is strongly inefficient. The duplication of resource in wasteful R&D implied by it may contribute to explain why the empirical link between investment in R&D and growth is not always so clear.

5) Last but not least, the policy variable T can be regarded as endogenous. The optimal patent T^* has not only to equilibrate gains from incentives to innovation and costs of monopolistic distortions, but it must also take into account

[6] *Proof*: $\bar{\pi}$ is approximately defined by $p(\bar{\pi}T) = 1/T - r$. It follows that $d\bar{\pi}/dr = -p'T < 0$ and $d\bar{\pi}/dT = -\left(p'\bar{\pi} + T^{-2}\right)/p'T < 0$. Q.E.D.

the endogenous decision of firms between patenting and hiding innovations and the effect of the patent on imitation decisions. This relationship is even more interestring if firms endogenously choose the size of their innovations since patent legislation will affect also this.

3 Persistent Monopolists

This section will study a model of patent races with different market structures based on Lee and Wilde (1980), Reinganum (1985) and Etro (2003), to show under which conditions innovation by leaders emerges.

Consider a market in which a monopolist with a patent on the leading edge technology is obtaining a flow of profits π, but a superior technology, if discovered, would give the right to a new patent. The patent race for the next technology involves n "entrant" firms $i = 1, ..., n$ and the incumbent patentholder, M. Each firm can participate in the patent race by paying a fixed cost F and hiring l^i workers at the market wage w, as to obtain an instantaneous probability of innovation:

$$h_i = h(l^i)$$

according to a standard Poisson process, where $h(0) = 0$, $h'(l) > 0$, $h''(l) < 0$ and we assume $\lim_{l \to 0} h'(l) > w(V-F)^{-1}$. The aggregate arrival rate of innovation will be the sum of the individual arrival rates of the n entrants plus the one of the incumbent:

$$p = \sum_{i=1}^{n} h(l^i) + h(l^I)$$

Let us assume that the innovation can be drastic or nondrastic. In the former case the winner will be the only monopolisitc producer with the new patent, in the latter, if the incumbent loses the patent race, a duopoly between the winner and the incumbent sets in. In general, the value of winning the patent race for the incumbent is denoted with V^W, while if an entrant wins, the previous incumbent obtains V^L and the winner obtains V^E, which is obviously smaller than V^W. The standard assumption is that, even if the innovation is drastic and the duopoly is characterized by perfect collusion, the sum of the discounted profits obtained by the two duopolists cannot be greater than the discounted profits obtained by the incumbent who wins the patent race:

$$V^W \geq V^E + V^L \tag{5}$$

Notice that the case of drastic innovations is a particular case for $V^W = V^E$ and $V^L = 0$.

Using the properties of Poisson processes in a standard fashion, the objective function of entrant i is:[7]

$$\Pi^i = \frac{h(l^i)V^E - wl^i}{\left[r + \sum_{j=1}^n h(l^j) + h(l^{M_i})\right]} - F \tag{6}$$

where r is the exogenous interest rate. The objective function of the incumbent monopolist is given by:

$$\Pi^L = \frac{h(l^M)V^W + \pi - wl^M + \sum_{j=1}^n h(l^j)V^L}{\left[r + \sum_{j=1}^n h(l^j) + h(l^M)\right]} - F \tag{7}$$

3.1 Cournot competition

Despite the features of the equilibrium under Cournot competition and free entry represent a kind of folk theorem in the literature, I am only aware of papers focusing on the case without free entry, so it seems worthwhile to summarize all the relevant results in this subsection. Hence I assume that each firm chooses the investment in R&D, taking as given the one of the others and the interest rate. Each entrant chooses l^i as to satisfy the first order condition:

$$\left[h'(l^i)V^E - w\right][r + \sum_{j=1}^n h(l^j) + h(l^M)] = h'(l^i)\left[h(l^i)V^E - wl^i\right] \tag{8}$$

which provides a unique best response function for the amount of hired workers (interiority is guaranteed by the Inada conditionon the function $h(\cdot)$). Straightforward differentiation shows that this best response is increasing in the expected value of innovation, the interest rate, the number of firms and the flow of investment of each other firm, implying strategic complementarity, while it is decreasing in the wage. Moreover, it can be shown that the uniqueness of the best response function for each firm implies equal investment between the entrants for a given number of firms. In other words, the equilibrium is symmetric between the entrants. The incumbent chooses l^M to satisfy the first order condition:

$$\left[h'(l^M)V^W - w\right][r+\sum_{j=1}^n h(l^j)+h(l^M)] = h'(l^M)\left[h(l^M)V^W + \pi - wl^M + \sum_{j=1}^n h(l^j)V^L\right] \tag{9}$$

[7] We may assume a more general cost function $c(wl) > 0$ with $c'(\cdot) > 0$ and $c''(\cdot) > 0$, in which case we would not need concavity of the $h(\cdot)$ function.

which defines a analogous best response function decreasing which does not necessarily need to be increasing in the investment of each entrant. Indeed:

$$\frac{dl^M}{dl^i} \propto h'(l^i)[h'(l^M)V^W - w] - h'(l^i)V^L h'(l^M)$$

is ambiguous. Anyway, this reaction function is decresing in the flow of current profits π and in V^L, which this implies that, *ceteris paribus*, the incumbent invests less than each entrant and has lower expected profits from participating in the patent race (Reinganum, 1983). Finally, the investment of all firms is increasing in r, V^W and n while decreasing in π, V^L and w.

Assuming free entry and noticing that the expected profit functions of all firms are decreasing in the number of firms, we can conclude that the incumbent will stop researching if the number of firms is great enough- this is the well known *Arrow effect* (Arrow, 1962, or replacement effect) - and the entrants will break even if the number of firms achieves a still higher bound. This bound is defined from the free entry condition:[8]

$$h(l)V^E - wl = F[r + nh(l)] \tag{10}$$

where I have used $l^M = 0$ and the symmetry of the equilibrium. Rearranging this equation, we can re-express the equilibrium flow of investment in the following implicit way:

$$h'(l)\left(V^E - F\right) = w \tag{11}$$

which is increasing in the difference between expected value of the innovation and fixed cost, decreasing in the wage rate and independent from the interest rate. The equilibrium number of firms is:[9]

$$n = \frac{V^E}{F} - \frac{wl}{Fh(l)} - \frac{r}{h(l)}$$

which turns out increasing in V^E, but decreasing in F and in the interest rate r. The effect of an increase in wage on the number of firms is however ambiguous.

As an example let us consider the Poisson arrival rate:

$$h(l) = \log(1 + l)$$

[8] In this paper I ignore, as usual, the integer constraint on n and consider it a real number, but generalizations are straightforward.

[9] With the more general cost function we would have:

$$h'(l)(V^E - F) = wc'(wl)$$

and

$$n = \frac{V^E}{F} - \frac{c(wl)}{Fh(l)} - \frac{r}{h(l)}$$

which satisfies our assumptions as long as $w < V^E - F$. With this specification, each firm employs:

$$l = \frac{V^E - F - w}{w}$$

researchers and the equilibrium number of firms is:

$$n = \frac{V^E}{F} - \frac{(V^E - F - w)}{F \log\left(\frac{V^E - F}{w}\right)} - \frac{r}{\log\left(\frac{V^E - F}{w}\right)}$$

In the relevant region (that is for $w < V^E - F$), the number of firms is an inverted U curve in the wage, decreasing in it for low wages and increasing in it for high wages. The turning point is at the wage $\hat{w}$ satisfying:

$$(V^E - F)\log(V^E - F) + (1 - r)F = (V^E - F)\log\hat{w} + (V^E - \hat{w})$$

Notice however that the aggregate arrivale rate of innovation in equilibrium is:

$$p = n\log(1 + l) = \frac{V^E}{F}\log\left(\frac{V^E - F}{w}\right) - \frac{(V^E - F - w)}{F} - r$$

which is always decreasing in the wage rate.[10]

3.2 Stackelberg competition

I will now drop the hypothesis of Nash behavior and will assume that the the patentholder has the opportunity to make a strategic precommitment to a level of employment of researchers. This may happen through a specific investment in R&D laboratories, by directly hiring researchers or in a number of other ways. Our strategic assumption seems a natural one since the patentholder can be easily seen in a different perspective from all other entrants in the patent race.

[10] As another example, consider the arrivale rate $h(l) = l$ associated with the cost function $c(wl) = (wl)^2/2$ which implies the equilibrium employment per firm:

$$l = \frac{V^E - F}{w}$$

and the equilibrium number of firms:

$$n = \frac{V^E}{2F} + \frac{1}{2} - \frac{rw^2}{V^E - F}$$

which is always decreasing in the wage rate. The aggregate arrival rate of innovations is:

$$p = nl = \frac{\left(V^E\right)^2 - F^2}{2Fw^2} - r$$

The opportunity to make a strategic precommitment is exploited by the incumbent as to increase its expected profits, but done so in dramatically different ways according to the competitive structure of the patent race. If the structure is characterized by a fixed number of firms, the incumbent leader will commit to a low level of investment because such a strategy will induce a reduction in the investment of the other firms and a longer expected lifespan of the current patent. But if entry in the patent race is free, the leader will commit to a high level of investment. Indeed, the investment of the leader perfectly crowds out that of the entrants, leaving constant the aggregate probability of innovation, as given by the free entry constraint. Hence the marginal cost of investment is lower for the leader than for the entrants, whose investment does affect the aggregate probability of innovation.

More formally, I will consider a two stage patent race. In the first stage, the leader chooses whether to participate in the new patent race and, in the former case, its employment l^M. In the second stage all the entrants choose their own investment l^i , knowing the investment of the leader and taking as given the investment of all other entrants. Obviously our equilibrium concept is subgame perfection with the entrants playing Nash in the second stage.

Let us consider the second stage. Each entrant chooses l^i to maximize their expected profits. Its choice satisfies the same first order condition as before, which implies a reaction function for l^i increasing in l^M, $\phi^i(l^M)$: the more aggressive the leader, the more aggressive the followers. In the first stage, the choice of the leader l^M satisfies the first order condition:

$$\left[h'(l^M)V^W - w + \frac{\partial\left[\sum_{j=1}^n h(l^j)\right]}{\partial l^M}V^L\right][r + \sum_{j=1}^n h(l^j) + h(l^L)] = \tag{12}$$

$$= \left[h'(l^M) + \frac{\partial\left[\sum_{j=1}^n h(l^j)\right]}{\partial l^M}\right]\left[h(l^M)V + \pi - wl^M + \sum_{j=1}^n h(l^j)V^L\right]$$

and we assume that the second order condition is satisfied.

3.2.1 Exogenous number of firms

Let us take as given the number of entrants n and assume that it is low enough that entry is actually profitable in equilibrium. The system (8)-(12) defines the equilibrium. Changes in r and π induce similar effects to the Cournot case. However, in general, the comparative statics of the investment of each firm with respect to w, n and V^k for $k = W, L, E$ is ambiguous. For instance, an increase in the number of entrants induces a direct positive effect on each firm's investment, but it also has an ambiguous effect on $\partial\phi^i(l^M)/\partial l^M$ and hence an indirect ambiguous effect on the investment of the leader. If the latter induces a net reduction of the investment of the leader, there is a further negative effect on the investment of the entrants in the second stage, which may overturn the initial effect. Paradoxically, an increase in the value of the innovation also makes the entrants more aggressive, and this may induce a reduction of the investment of the leader, with ambiguous consequences.

Indeed, symmetry between the entrants in the second stage implies the equilibrium system:

$$f(\cdot) \equiv \left[h'(l)V^E - w\right]\left[r + nh(l) + h(l^M)\right] - h'(l)\left[h(l)V^E - wl\right] = 0$$

$$\begin{aligned} g(\cdot) \equiv \left[h'(l^M)V^W - w + \tfrac{\partial nh(l)}{\partial l^M}V^L\right]\left[r + nh(l) + h(l^M)\right] + \\ - \left[h'(l^M) + \tfrac{\partial nh(l)}{\partial l^M}\right]\left[h(l^M)V + \pi - wl^M + \tfrac{\partial nh(l)}{\partial l^M}V^L\right] = 0 \end{aligned}$$

with:

$$\frac{\partial nh(l)}{\partial l^M} = \frac{\partial n}{\partial l^M}h(l) + nh'(l)\phi'(l^M)$$

where:

$$\phi'(l^M) = \frac{-\left[h'(l^M)V^E - w\right]h'(l^M)}{h"(l)\left[V^E\left(r + (n-1)h(l) + h(l^M)\right) + wl\right]} > 0$$

Since $\partial\phi'(l^M)/\partial r < 0$, $\partial\phi'(l^M)/\partial\pi = 0$ and $\partial\phi'(l^M)/\partial n > 0$, while $\partial\phi'(l^M)/\partial V^E$ is ambiguous, by totally differentiating the system above we obtain the comparative statics for $x = r, \pi, w, n, V$:

$$\begin{bmatrix} \frac{dl}{dx} \\ \frac{dl^M}{dx} \end{bmatrix} = \frac{-1}{\Delta}\begin{bmatrix} g_{l^M} & -f_{l^M} \\ -g_l & f_l \end{bmatrix}\begin{bmatrix} f_x \\ g_x \end{bmatrix}$$

where $\Delta \equiv f_l g_{l^M} - f_{l^M} g_l > 0$ by assumption of stability, and $f_l < 0$ (from the assumption of stability), $f_{l^M} > 0$, $f_r > 0$, $f_\pi = 0$, $f_n > 0$, $f_{V^E} > 0$, $g_l > 0$, $g_{l^M} < 0$, $g_r > 0$, $g_\pi < 0$ and f_w, g_w, g_n and g_{V^k} for $k = W, L, E$ have ambiguous signs. It follows that comparative statics for n, w and V^k is ambiguous, but $dl^M/dr > 0$, $dl/dr > 0$, $dl^M/d\pi < 0$, and $dl/d\pi < 0$. Summarizing, *Stackelberg competition for a given number of firms implies an investment for each firm which is increasing in the interest rate and decreasing in the flow of current profits, but ambiguously dependent on the wage, on the value of the innovation and the number of firms.*

Moreover, Stackelberg competition for a given number of firms implies an aggregate investment in R&D which is increasing in r and decreasing in π, and an expected lifespan of the current patent which is decreasing in r and increasing in π, while the effects of changes in V and n are ambiguous. Finally, we can prove that Stackelberg competition induces less aggregate investment in R&D than Cournot competition by each firm. Graphically, the reaction function of the representative entrant in the space (l^M, l) is upward sloping. The Cournot equilibrium is at the intersection of this with the reaction function of the leader, while the Stackelberg equilibrium is at the tangency of the the lowest isoprofit locus of the leader with the reaction function of the representative entrant. It is easy to verify that in any case cases the leader invests less than the representative entrant, but in the Stackelberg case, the leader invests even less then in the Cournot case and less than each other single firm. Moreover the representative entrant invests less in the Stackelberg equilibrium than in the Cournot equilibrium. Since the number of entrants is given, it must be that the aggregate investment is reduced. Summarizing, *Stackelberg competition for a given number of firms implies a) a lower investment than Cournot competition for all the entrant firms and the leader patentholder, a smaller aggregate investment, a longer expected lifespan of the current patent, and b) lower investment for the leader patentholder than for each of the other firms.*[11]

The intuition is that by investing more the incumbent rises the incentives to invest for each entrant, since investment is characterized by strategic complementarity in this environment, and this both increases the probability of an innovation by the follower, and reduces the expected lifespan of the current patent. Hence, the incumbent uses its first mover advantage to reduce its own investment. A similar argument is advanced in a more general set of games by Fudenberg and Tirole (1984), Dixit (1987).[12] Stackelberg leadership with a fixed number of firms due to some barriers to entry does not give a new rationale for incumbents' investment in R&D. Actually the opposite happens!

[11] *Proof*: Notice that:

$$\frac{\partial \left[\sum_{j=1}^{n} h(l^j)\right]}{\partial z_L} = nh'(l)\phi'(l^M) > 0$$

is the only element that differentiates the system (8)-(12) from the system (8)-(**??**). Since it is positive it is clear that the marginal cost of investment for the leader is higher (since it induces greater investment by the outsiders), and thus the leader invests less for a given investment from the outsiders. Strategic complementarity ($\phi'(l^M) > 0$) implies that the same must be true also for the outsiders in equilibrium, which proves a). A graphical proof of b) is suggested in the text. Q.E.D.

[12] The study of Stackelberg competition in quantities with barriers to entry is undertaken by Fudenberg and Tirole (1984) and especially Dixit (1987), where they show how strategic complementarity and substituibility determine whether the leader is going to be less or more aggressive than the entrants. Etro (2002) extends their general framework to the free entry case, showing that the leader is always more aggressive when there is free entry. An example of this general result is applied in this paper when I assume free entry.

3.2.2 Endogenous number of firms

Let us now consider the free entry case, in which the leader has to foresee the effects of its investment choice on the equilibrium number of entrants. Despite this complication, it turns out that it is quite easy to characterize the new equilibrium. In the second stage all the entrant firms choose the same flow of investment l. Using this symmetry, the zero profit condition becomes:

$$h(l)V^E - wl = F[r + nh(l) + h(l^M)] \tag{13}$$

Substituting (13) in the equilibrium first order condition of the entrants (8) we obtain an implicit expression for the entrant's investment:

$$h'(l)\left(V^E - F\right) = w$$

which provides (11) again and does not depend on the leader's decision.[13] However, the equilibrium number of firms, given by:

$$n = \frac{V^E}{F} - \frac{wl}{h(l)F} - \frac{r + h(l^M)}{h(l)} \tag{14}$$

does depend on the leader's choice. Totally differentiating this condition - and using the fact that l does not depend on l^M - we can obtain the expected change of investment in R&D of each entrant for a change in the leader's investment:

$$\begin{aligned} \frac{\partial\left[\sum_{j=1}^{n} h(l^j)\right]}{\partial l^M} &= \frac{\partial nh(l)}{\partial l^M} = \frac{\partial n}{\partial l^M}h(l) + nh'(l)\frac{\partial \phi(l^M)}{\partial l^M} = \\ &= -h'(l^M) \end{aligned}$$

which has the opposite sign of the case without free entry. Despite an increase in the investment of the leader increasing the investment of each entrant, the effect on the equilibrium number of firms is negative and large enough to more than compensate the former. Substituting in (12) we obtain an implicit expression for the leader investment:

$$h'\left(l^M\right)(V^W - V^L) = w \tag{15}$$

[13] The reason for this independence is subtle, but generalizes to a large class of games with Stackelberg competition in quantities and free entry - as long as the profit function of each firm depends on the investments of the other firms through the sum of some function of the those investments (Etro, 2002). Suppose starting in an equilibrium without free entry. An increase in the leader's investment has two effects, a direct one and an indirect one. The first is to increase the investment of an entrant, but it also decreases the number of profitable entries. Since the investment of the entrant is positively related to the number of firms, the second indirect effect is to decrease the investment of an entrant. These two effects cancel each other out under free entry.

In other words, the leader chooses:

$$\begin{aligned} l^{MS} &= argmax\left[\frac{h(l^M)V^W+\pi-wl^M+n^S h(l^S)V^L}{[h(l^S)V^E-wl^S]}-1\right]F= \\ & argmax\left[h(l^M)V^W+\pi-wl^M+nh(l^S)V^L\right] \end{aligned}$$

with first order confition:

$$h'(l^M)V^W=1-\frac{\partial n^S h(l^S)}{\partial l^M}V^L$$

but $\frac{\partial n^S h(l^S)}{\partial l^M}=-h'(l^M)$ so in equilibrium we have (15). Clearly, condition (5), always implies that $l^{MS}>l^S$.

The Arrow effect disappears. The investment of the leader is directly related to the net perspective value of innovating V^W-V^L, which is strictly higher than the one of the entrant V^E. This induces a stronger persistence of monopoly in the case of nondrastic innovations than in the case of drastic innovations. The equilibrium also implies a lower number of entrants than in Cournot equilibrium with free entry. It follows that *Stackelberg competition with free entry implies a) the same investment as Cournot competition for the entrant firms with a lower number of entrants and b) a higher investment for the leader patentholder than for each of the other firms.*

Stackelberg competition with free entry induces the aggressive behavior of the monopolist in the patent race: while under Cournot competition the incumbent was not doing any research, the first mover advantage delivers a strong incentive to invest for the incumbent. The intuition is related to the perception the leader has of the entry process. It is understood that any profitable opportunity for doing R&D left open by the leader will be seized by new entrants until their profits are zero. The aggregate probability of innovation is determined by the free entry constraint independently of the investment of the leader and is thus taken as given by the same leader. So, the monopolist loses the strategic incentive to keep its investment low: the latter is not going to affect the expected lifespan of the current patent. Hence, the only purpose of investing in R&D for the leader is to actually win the patent race, and the incentives to do it are now higher than those of any other entrant. An intuitive way to see this asymmetry again depends on the fact that the leader is taking as given the aggregate probability of innovation; so, the optimal investment of the leader maximizes $h(l^M)(V^W-V^L)-wl^M$ without taking into account the impact on the aggregate arrival rate of innovation. This impact, instead, is taken into account by each entrant and reduces their profits, explaining why entrants invest less than the leader. Notice that the Arrow effect does not play any role in this equilibrium and the investment of the leader is independent from the current flow of profits: under Stackelberg competition the Arrow effect disappears.

It is easy to derive that *Stackelberg competition with free entry implies: a) an investment for each entrant firm which is increasing in the value of the innovation V^E and decreasing in the wage and the fixed cost, while independent from interest rate and current profits, b) an investment for the incumbent leader which is increasing in the net value of the innovation $V^W - V^L$, decreasing in the wage and independent from other factors, c) a number of firms and an aggregate investment which decrease in the interest rate, are independent from profits and are ambiguously affected by wage, value of the innovation and fixed costs and d) an expected lifespan of the current patent which is increasing in the fixed cost and the wage and decreasing in the value of innovation, while it is independent from interest rate and profit flows.*

Finally let us consider our example with $h(l) = \log(1 + l)$. With this specification, the leader employs:

$$l^M = \frac{V^W - V^L - w}{w}$$

researchers and the equilibrium number of firms is:

$$n = \frac{V^E}{F} - \frac{(V^E - F - w)}{F \log\left(\frac{V^E - F}{w}\right)} - \frac{r}{\log\left(\frac{V^E - F}{w}\right)} - \frac{\log\left(\frac{V^W - V^L - w}{w}\right)}{\log\left(\frac{V^E - F}{w}\right)}$$

3.2.3 Implications

Here I will summarize the main implication of the model.

1) If we believe that Stackelberg competition is the right assumption in the study of patent races - which may be true in some specific contexts but not in all and is ultimately an empirical issue -, we obtain some sharp conclusions from this analysis. Paradoxically, if we see a market for innovation dominated by the current monopolist and another one in which the current monopolist does not research (or does less research than the other entrants), the former is competitive, while the latter is not: in other words *a market characterized by high persistence of monopoly is competitive, while one with systematic leapfrogging must be characterized by some barriers to entry*! This is exactly the opposite conclusion than the one we obtain by assuming Cournot competition.

2) The model also implies that aggregate investment in R&D may be higher under Stackelberg competition, while the number of firms is typically higher under Cournot competition. From a welfare point of view, since the externality

associated with Nash behavior, the business stealing effect and the consumer-surplus effect (due to the positive difference between social and private value of innovations) work in different directions, we cannot compare different equilibria, but it may well be the case that different policy prescriptions characterize the two equilibria. Only a general equilibrium approach would allow a proper comparison.

3) The model easily generalizes in a general equilibrium environment as shown by Etro (2001) where I extend the Schumpeterian growth model of Aghion and Howitt (1992) to patent races as those described above and study the consequences of persistence of monopolistic positions on growth.

4 Radical Innovators

Are outsiders'innovations more radical? A widespread view claims that this is the case since patentholders may have a technological advantage in obtaining small improvements on their technologies, so as to induce entrants to try replacing the patentholder with radical innovations. In this section we question this view studying the case in which the size of innovations is endogenous.

Each firm can invest in two different ways: on one hand the firm can invest as to increase the probability of innovation, and on the other, the firm can invest as to obtain a greater profit from the innovation in the case the patent is obtained. The latter investment is also characterized by decreasing returns to scale. Without it, participation in the patent race is pointless, because in case of victory, the prize is negligible; in other words the two investments are complementary.

The value of the innovation is now a function of a firm specific investment, $V(x^i)$ with $V(0) = 0$, $V'(x^i) > 0$ and $V''(x^i) < 0$ where x^i is a flow of investment which is necessary to discover an innovation of quality $V(x^i)$. The objective function of firm k becomes:

$$\Pi^k = \frac{h(l^k)V(x^k) + \pi^k - wl^k - x^k}{\left[r + \sum_{j=1}^n h(l^j) + h(l^M)\right]} - F \tag{16}$$

where $\pi^k = 0$ for any entrant $k = 1, ..., n$ and $\pi^k = \pi$ for the incumbent M. In the free entry equilibrium with Stackelberg competition we have:

$$h'\left(l^S\right)\left[V(x^S) - F\right] = w \qquad h\left(l^S\right)V'(x^S) = 1$$

$$h'\left(l^M\right)V(x^{LS}) = w \qquad h\left(l^M\right)V'(x^{MS}) = 1$$

$$n^S = \frac{V(x^S)}{F} - \frac{wl^S}{h(l^S)F} - \frac{r + h(l^{MS})}{h(l^S)}$$

and the previous results go through. By totally differentiating the above system of two conditions determining l^S and x^S, we have:

$$\left[\begin{array}{c} \frac{dl}{dF} \\ \frac{dx}{dF} \end{array}\right] = \frac{1}{\Delta(F)} \left[\begin{array}{cc} h(l)V''(x) & -h'(l)V'(x) \\ -h'(l)V'(x) & h''(l)[V(x) - F] \end{array}\right] \left[\begin{array}{c} h'(l) \\ 0 \end{array}\right]$$

where $\Delta(F) = h''(l)[V(x) - F]h(l)V''(x) - [h'(l)V'(x)]^2$, hence the comparative statics depends on the sign of $\Delta(F)$. However, $\Delta(0)$ is the determinant of the maximization problem for the leader, which we assume positive for the solution to be interior. By continuity, there exists a right neighborood of $F = 0$ for which $\Delta(F) > 0$ and:

$$\frac{dl}{dF} = \frac{h(l)h'(l)V''(x)}{\Delta(F)} < 0$$

$$\frac{dx}{dF} = -\frac{h'(l)^2 V'(x)}{\Delta(F)} < 0$$

Moreover, we have:

$$\frac{dl}{dw} = \frac{h(l)V''(x)}{\Delta(F)} < 0$$

$$\frac{dx}{dw} = -\frac{h'(l)V'(x)}{\Delta(F)} < 0$$

hence also an increase in wage reduces both investments in R&D as long as F is small enough.

Summarizing, it follows that *Stackelberg competition with free entry and an endogenous amount of innovation implies a) the same investment and value of innovation as Cournot competition for the entrant firms, positive investment for the leader patentholder, a lower number of entrants and b) a higher investment for innovations of greater value for the leader patentholder than for each of the other firms if the fixed cost is small enough. Both kinds of investments are decreasing in the wage and the fixed cost.*

Notice that for small fixed costs, the incumbent invests in innovations of higher quality. Indeed, each firm equalizes the marginal productivity of the two investments, which end up being complementary. That is why our traditional incentive to invest a lot as the leader due to the absence of the Arrow effect also induces investment in innovations of greater value: the two forces strenghten each other.

5 Conclusion

In this paper I have developed different theories of the value of innovations which are consistent with important facts as the fact that some innovations are kept secret, that monopolist patentholders often incur in relevant investments in R&D and often they invest for more radical innovations than the outsiders. This scope of the paper was to point out the complexity of the choices of firms racing for innovations. These and others should be focused in future research investigating their consequences for growth policies and patent laws.

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