

# Persistence of Monopolies and Research Specialization

Philipp Weinschenk\*

Bonn Graduate School of Economics

and Max Planck Institute for Research on Collective Goods

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We examine the persistence of monopolies in markets with innovations when the outcome of research is uncertain. We show that for low success probabilities of research, the incumbent can seldom preempt the potential entrant. Then the efficiency effect outweighs the replacement effect. It is vice versa for high probabilities. Moreover, the incumbent specializes in “safe” research and the potential entrant in “risky” research. We also show that the probability of entry is an inverted U-shape in the success probability. Since even at the peak entry is rather unlikely, the persistence of the monopoly is high.

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\*Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, 53113 Bonn, Germany, weinschenk@coll.mpg.de. I thank Philippe Aghion, Felix Bierbrauer, Mathias Dewatripont, Christoph Engel, Paul Heidhues, Martin Hellwig, Fabian Herweg, Jos Jansen, Stephan Lauer mann, Daniel Müller, Tymofiy Mylovanov, Nora Szech, as well as seminar participants at the Max Planck Institute for Research on Collective Goods in Bonn, the Summer School on Contract Theory in Bronnbach, and the University of Bonn for helpful comments and suggestions.

## 1 Introduction

What forces influence the persistence of monopolies in markets with innovations? The two seminal papers come to different conclusions. In Gilbert and Newbery (1982; “GN”) the efficiency effect determines the outcome, whereas in Reinganum (1983; “RE”) it is the replacement effect. The efficiency effect predicts that the incumbent’s incentive to remain a monopolist through innovating is at least as great as the entrant’s incentive to become a duopolist. In contrast, the replacement effect, also known as Arrow (1962) effect, says that the entrant’s incentives to innovate are higher than the incumbent’s, since the latter takes into account that its existing technology will be replaced, while the former does not.<sup>1</sup> We build a unifying model.

In our one shot, two firms model, the first moving incumbent may be able to preempt, meaning that the incumbent may be able to discourage the potential entrant from investing in research by investing itself.<sup>2</sup> The paper allows for uncertainty with respect to the outcomes of innovative activities. Because of this uncertainty, preemption is less than perfect. For high success probabilities we get a result in the spirit of GN: preemption is (almost) perfect, and so the efficiency effect plays the major role. Intuitively, the potential entrant’s expected profit from research greatly depends on the incumbent’s research decision, and so it is very likely that the incumbent can and does preempt the potential entrant. For low success probabilities the same arguments apply in reverse. Then the result is à la RE.

A corollary of the preceding insight is that research with a high success probability is more likely done by the incumbent than by the potential entrant. So incumbents specialize in “safe” research, and potential entrants in “risky” research. We also show that the reverse is true: potential entrants’ research is “riskier” than incumbents’. Moreover, the probability of entry is an inverted U-shape in the success probability of research. Since even at the peak of the inverted-U the probability of entry is at most 1/4, the persistence of monopoly is high.

The research process considered by GN is commonly interpreted as an auction. As an extension, we integrate such an auction process into our model. This changes our results: regardless of the success probability, the incumbent will always outbid the entrant if the innovation is non-drastic. So entry will never occur. This is exactly GN’s result, in a more general approach that allows for uncertainty of the research process. Intuitively, preemption is perfect, since whenever the incumbent wins the

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<sup>1</sup>The differences between GN and RE lead to a debate; see Reinganum (1984) and Gilbert and Newbery (1984).

<sup>2</sup>The idea that a dominant firm might use its investment decision as a strategic device to persuade a potential entrant not to enter stems from Spence (1977) and Dixit (1980). They consider capacity investments.

## 2 The Model

auction, the entrant necessarily loses it.<sup>3</sup>

Denicolo (2001) and Etro (2004) consider a research process of the RE type where the replacement effect disappears, since the aggregate R&D effort is independent of the incumbent's decision. Harris and Vickers (1985), and Doraszelski (2003) also incorporate learning effects in the research process. The literature on the persistence of monopoly in markets with innovations is surveyed by Gilbert (2006).

Our paper is also related to the literature which explains why incumbents and entrants differ in the riskiness of the research they do. Kihlstrom and Laffont (1979) look at differences in the risk-attitudes of firms, De Meza and Southey (1996) at excessive optimism of entrepreneurs. Scherer and Ross (1990, ch. 17) blames the bureaucracy in large companies. Baumol (2004) highlights educational differences between researchers in incumbent firms and entrepreneurs that engage in research. In Rosen (1991) the ex ante high-cost firm must spend more than the ex ante low-cost firm to yield the same cost level. Through this asymmetry, the former chooses a riskier research project than the latter. We offer a new explanation. Moreover, beside sequential investment decisions, the firms in our model differ only in one aspect: the incumbent is already on the market, whereas the potential entrant may enter it.

The model is presented, analyzed, and discussed in Sections 2, 3, and 4, respectively. An auction setting is considered in Section 5. Section 6 concludes. Proofs are in the Appendix.

## 2 The Model

There are two risk neutral firms, an incumbent  $I$ , and a potential entrant, also called rival,  $R$ . Each firm decides sequentially whether or not to invest in a research project with success probability  $p \in (0, 1]$ . The expected costs of investing are  $z > 0$ . Whether each firm's project is successful or not is independently determined by the nature. A successful firm gets a process innovation that enables it to produce the homogeneous good at per-unit-costs of  $\underline{c}$ . If  $I$  does not invest or its project fails, it can produce at per-unit-costs of  $\bar{c}$  by using its old technology, where  $\bar{c} > \underline{c} > 0$ . In contrast, if  $R$  does not invest or its project fails, it cannot produce at all. The firms' production costs determine their profits in the last stage, where they compete à la Bertrand. A firm's payoff is its Bertrand profit minus its investment. The timing is

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<sup>3</sup>This is also the intuition for GN. The intuition for RE is as follows: It is assumed that the innovation is drastic (or almost drastic), and so both firms yield the same (almost the same) prize when they win the race. The incumbent is more happy with the status quo than the potential entrant, since under the status quo only the former receives a positive flow profit. Hence the entrant is eagerer to change the status quo and so invests in equilibrium more than the incumbent.

### 3 Analysis

as follows:

0. Parameters are drawn.
1.  $I$  decides whether to invest in research.
2.  $R$  decides whether to invest in research.
3. Nature determines whether the investment of *each* firm is successful or not.
4. Bertrand competition. Payoffs are made.

There is perfect information and firms cannot collude. The solution concept is subgame perfect Nash equilibrium.

#### Bertrand Profits

The Bertrand profit of firm  $J$  is denoted  $\pi(c_J, c_{-J})$ , where  $c_J, c_{-J} \in \{\bar{c}, \underline{c}, -\}$ .<sup>4</sup> The symbol “ $-$ ” indicates when a firm cannot produce at all. Tirole’s (1988) analysis shows that the following lemma holds, were the maximal profit  $\pi(\underline{c}, -)$  is normalized to 1.<sup>5</sup>

#### Lemma 1:

- (i)  $\pi(-, \bar{c}) = \pi(-, \underline{c}) = \pi(\underline{c}, \underline{c}) = \pi(\bar{c}, \underline{c}) = 0$ ,
- (ii)  $0 < \pi(\bar{c}, -) < 1$ ,
- (iii)  $1 \geq \pi(\underline{c}, \bar{c})$ ,
- (iv)  $1 - \pi(\bar{c}, -) < \pi(\underline{c}, \bar{c})$ .

When a firm has per-unit-costs that are not lower than its competitor’s, its Bertrand profit is zero, see (i). (ii) states that a monopolistic incumbent is strictly better off with lower per-unit-costs, but with high per-unit-costs it still makes a positive profit. (iii) is the *efficiency effect*:  $I$ ’s incentive to remain a monopolist through innovating is at least as great as  $R$ ’s incentive to become a duopolist. However, when  $I$  takes into account that its old technology is replaced when it innovates the inequality of (iii) reverses, see (iv). This is the *replacement effect*, which predicts that  $R$ ’s incentives to innovate are higher than  $I$ ’s. Note, when  $\pi(\underline{c}, \bar{c}) = 1$  the innovation is called *drastic*.

### 3 Analysis

#### 3.1 Research Decisions

Firm  $J$ ’s research decision is denoted by  $a_J \in \{0, 1\}$ , where  $J \in \{I, R\}$ , and 0 (1) is (no) investment. A firm invests if and only if its expected payoff from doing so is

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<sup>4</sup>Since firms produce a homogeneous good and compete à la Bertrand,  $\pi$  is symmetric.

<sup>5</sup>Blume (2003) shows that when firms have different marginal costs, there also exist mixed-strategy Nash equilibria, where one firm plays a weakly dominated strategy. These equilibria are not considered.

### 3 Analysis

higher than from not investing.<sup>6</sup>

#### Rival's Research Decision

$R$ 's best response  $b_R$  depends on  $I$ 's action. Obviously,  $b_R[a_I = 0] = 1$  if and only if

$$z < \pi(\underline{c}, \bar{c})p; \quad (1)$$

$b_R[a_I = 1] = 1$  if and only if

$$z < \pi(\underline{c}, \bar{c})(1 - p)p. \quad (2)$$

So investing yields a higher expected payoff than not investing when the costs  $z$  are low enough.

#### Incumbent's Research Decision

$I$ 's best response  $b_I$  depends on  $R$ 's best response set, i.e., on  $R$ 's strategy. Straight-forward calculations yield that  $b_I[b_R(\cdot) = 0] = 1$  if and only if

$$z < (1 - \pi(\bar{c}, -))p; \quad (3)$$

$b_I[b_R(\cdot) = 1] = 1$ , if and only if

$$z < (1 - \pi(\bar{c}, -))(1 - p)p; \quad (4)$$

$b_I[b_R(a_I = 1) = 0, b_R(a_I = 0) = 1] = 1$  if and only if

$$z < p. \quad (5)$$

Note that it can never occur that  $R$  only invests if  $I$  also does. This implies that firms research decisions are strategic substitutes.

#### Equilibrium Research Decisions

Figure 1 shows how the firms' inequalities combine to determine the equilibrium research decisions, which we denote by the vector  $a^* = (a_I^*, a_R^*)$ . For parameter combinations on a line, the respective weak inequality holds with equality; for combinations below the line, the inequality is strict. The construction of the equilibria is straightforward. As can be seen, the parameter space is partitioned in four areas in which the equilibrium is unique and differs from the other areas. Moreover, every possible equilibrium occurs. The following Lemma summarizes formally.

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<sup>6</sup>Assumption A1 will guarantee that cases of indifference have a measure of zero. So it does not matter whether or not firms invests in case of indifference.



### 3 Analysis

invest, irrespective of what  $I$  has done, but where  $I$  is no longer motivated to invest, given that  $R$  will invest. For this set  $a^* = (0, 1)$ . As explained before, when  $p$  is high,  $R$ 's expected profit from research – and so its willingness to invest – is very sensitive upon  $a_I$ . Hence the replacement effect loses its power when  $p$  is high. In the extreme case of  $p = 1$ , it has no power at all, since  $R$  never invests in research when  $I$  has invested.

When the costs are very low, all inequalities are satisfied and so  $a^* = (1, 1)$ . In the remaining parameter set, costs are so high that  $a^* = (0, 0)$ .

#### 3.2 Results

From Figure 1 it is intuitive that for low  $p$  values we get results close to RE, namely that the replacement effect is more important than the efficiency effect, whereas for high  $p$  values it the other way round and so we have results close to GN. We formalize this thought by integrating Stage 0 in our model, where first  $p$  is drawn from density  $g$ , and then  $z$  is drawn from conditional density  $h$ .<sup>7</sup>

We have to make some assumption on  $h$  to yield concrete results.

**Assumption A1:**  $h(z|p)$  is uniform in  $z$  for  $(z, p) \in \mathcal{S}$ .

**Proposition 1:** Suppose that A1 holds.

- (i) When  $p$  is low enough, it is more likely that the replacement effect determines the outcome than that the efficiency effect does. It is vice versa when  $p$  is high enough.
- (ii) The probability of entry is at most  $p(1 - p)$ .

Intuitively, given that a high  $p$  was drawn, the probability that  $z$  will lie in the small interval where  $a^* = (0, 1)$  is smaller than that it will lie in the large interval where  $a^* = (1, 0)$ . For a low  $p$  value it is the other way round. The reason why the relative sizes of the intervals crucially depends upon  $p$  was described before.

The intuition for Proposition 1(ii) is as follows. When  $p$  is very low, research is seldom successful, and so entry rarely occurs. The latter is also true when  $p$  is very high. Then only for a small interval of costs does  $I$  not preempt  $R$ . So the probability of entry is roughly an inverted U-shape in  $p$ . Ignoring the probability mass outside  $\mathcal{S}$  (this is possible when one redefines the densities  $g$  and  $h$  accordingly) we get exactly an inverted U-shape. At the peak, the probability of entry is  $1/4$ . Hence, the probability that a monopoly persists is at least  $3/4$ .

Since the efficiency effect plays in favor of  $I$ , whereas the replacement effect favors  $R$ , the following corollary to Proposition 1(i) is obvious.

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<sup>7</sup>When  $z$  is drawn before or simultaneously to  $p$ , we can ignore the  $z$  value until  $p$  is drawn, and so preserve the vision that  $p$  is drawn first. Integrating Stage 0 is a kind of comparative statics analysis. We determine how important the different equilibria are, conditional on  $p$ .

**Corollary 1:** When  $p$  is low enough, it is more likely that  $R$  invests in research than that  $I$  does. So low  $p$  research is more likely done by  $R$  than  $I$ . It is vice versa when  $p$  is high enough.

Taking another view by looking at a large number of  $I$ - $R$ -pairs, Corollary 1 says that incumbents specialize in “safe” research, and potential entrants in “risky” research. Under some conditions, the reverse is also true.<sup>8</sup>

**Proposition 2:** Suppose that A1 holds and that  $g(p)$  has full support and is finite  $\forall p \in (0, 1]$ .  $R$ 's research fails more likely than  $I$ 's.

In other words, potential entrants' research is “riskier” than incumbents'.

## 4 Discussion

### 4.1 Empirical Relevance

That “risky” research is most often done by entrepreneurs and not by incumbents, and it is vice versa for “safe” research, is found by Baumol (2004). This fits Propositions 1(i). See also Scherer and Ross (1990, p. 653) or Chandy and Tellis (2000) for supporting evidence.

Astebo (2003, p. 227) finds that

the average probability that an independent inventor succeeds in commercialising his/her invention is estimated to about 0.07. In comparison, the probability of commercial success of conducting R&D in established firms is approximately 0.27,

where the later value is from Mansfield et al. (1977). This suits Proposition 2. Also the findings of Baumol (2004), and Bianchi and Henrekson (2005, p. 367) support this result.

Empirically, the persistence of monopolies seems to be high (Geroski 1995), as our model predicts, see Proposition 1(ii).

### 4.2 Other Distributions

Scherer and Ross (1990) elucidate how the costs of R&D are distributed in reality. Since we use expected costs and normalize, we cannot use their insights. So we admit, that we have absolutely no suspicion about how the “real” density  $h$  might look like. So A1 may seem very restrictive and crucial. This is not true. First,

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<sup>8</sup>Also a weaker assumption is sufficient for Proposition 2:  $g(p)$  is positive for at least two  $p$  values or sets with positive probability mass.

A1 assumes that  $h(\cdot)$  is uniform in  $z$  for parameters of the set  $\mathcal{S}$ , but it may still depend upon  $p$ . Second, A1 offers a *sufficient* condition. Third, when only the average density of  $h(\cdot)$  in the different equilibrium sets of  $\mathcal{S}$  is the same,<sup>9</sup> the proofs, and so also our results, stay unchanged.

### 4.3 The Research Process

The research process we have in mind is the following: an incumbent and a potential entrant have a firm-specific idea about how to achieve the non-firm-specific innovation. To find out whether an idea is realizable, a firm must engage in research. However, whether an idea is realizable or not is already determined by the nature. Hence it cannot be influenced by a firm – the success probability of research is exogenous. Moreover, if after research it turns out that an idea is not realizable, there is no way for that firm to get the innovation. Research has ultimately failed. Because of this, the game is not repeated as, for example, RE's is.

Instead of assuming that research fails technologically, we can also assume that it fails economically: with probability  $1 - p$ , the project needs additional funding which exceeds the economic value of the innovation.<sup>10</sup> This raises a more general idea, namely that one can understand the extremely simple research process we have specified as a reduced form of a more complex one.

The research process we consider is extremely discontinuous, firms can invest in research or not.<sup>11</sup> This is justified, since “research is believed to be both *more uncertain* and *more discontinuous* than development” (La Manna, 1994, p. 1424, italics supplied).

### 4.4 The Timing

When the incumbent does not decide before the rival about investing, preemption is not possible, and so the equilibria in the investment game partially change. From a purely theoretical point of view, the sequential move assumption of the firms is arbitrary. However, the assumption is not unusual in the literature and also has an

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<sup>9</sup>More technically this means that  $\forall p \in (0, 1]$ ,  $\bar{h}_{a^*=(1,0)}(p) = \bar{h}_{a^*=(0,1)}(p) = \bar{h}_{a^*=(1,1)}(p)$ , where  $\bar{h}_{a^*=i}(p)$  is the average density for the equilibrium of type  $i$ , given  $p$ . Additionally, one can easily show that Proposition 1(i) holds under the very mild assumption that  $\bar{h}_{a^*=i}(p)$  has an infimum and a supremum which are in  $\mathbb{R}^{++} \forall i \in \{(1, 0), (0, 1), (1, 1)\}$  and  $\forall p \in (0, 1]$ . Then also the essence of Proposition 1 (ii) holds: The probability of entry approaches 0, as  $p$  approaches 0 or 1. This is also true under the more technical, but even less restrictive assumption that  $\bar{h}_{a^*=i}(p)/\bar{h}_{a^*=j}(p)$  is finite  $\forall i, j \in \{(1, 0), (0, 1), (1, 1)\}$  and  $\forall p \in (0, 1]$ .

<sup>10</sup>Lukach, Kort, and Plasmans (2007) consider a model where two steps are needed to complete research, and after the first stage the incumbent learns about the costs of the second stage. In contrast to our model, the incumbent decides twice, and the potential entrant cannot research.

<sup>11</sup>This is the formulation of La Manna (1994), who considers different questionings and also includes a development stage.

intuitive appeal. First, the potential entrant might, in contrast to the incumbent, need some time to get information about the market or to get funding. Second, the incumbent's activity is visible for everyone, whereas the entrepreneur's activity is not. Third, preemption is a relevant real-world phenomenon and so our model should be able to capture it. According to Scherer and Ross (1990, p. 654), "Many cases can be found in which the threat of entry through innovation by a newcomer stimulated existing members to pursue well-known technical possibilities more aggressively."

### 4.5 Policy Implications

From Lemma 2, it is conspicuous that  $R$ 's incentives determine whether at least one firm does research or not. (1) is the relevant condition. Consider a research subsidy to  $R$  by a government who wants to increase the research activity. The subsidy can be translated into unchanged research costs and a higher  $\pi(c, \bar{c})$ . Graphically, the (1) line rotates upward. Consequently projects which were left unexplored are now explored. Surprisingly, the subsidy can be effective even if it is not paid. This is true whenever  $I$  preempts. A research subsidy to  $I$  increases the parameter set where both firms engage in research. However, whenever this subsidy is effective, it must be paid. In general, without subsidies, the presence of  $R$  has a nonnegative effect on the overall research activity, since without  $R$ ,  $I$  would invest in research only when (3) is satisfied.<sup>12</sup>

### 4.6 Are RE and GN comparable to our model?

The models of RE and GN depict patent races, but they can also be interpreted as preemption models: suppose that a firm that has successfully innovated (i) can keep its innovation secret, or (ii) there is a patent system that does not allow imitations, but allows that both firms innovate and use their innovations.<sup>13</sup> Then, when there is Bertrand competition,<sup>14</sup> a firm no longer *wants* to engage in R&D when its competitor was already successful, since it can no longer hope to make a profit with an innovation, although it still *can* engage in R&D.<sup>15</sup> In the conclusion we compare our model to RE's and GN's.

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<sup>12</sup>Dasgupta and Stiglitz (1980) yield similar effects in their model: "the threat of competition *may* lead the monopolist to engage in significantly faster research than it otherwise would" (p. 26, italics supplied).

<sup>13</sup>That two firms which innovate on the same market nonetheless innovate something different is quite likely.

<sup>14</sup>RE's model does not change when one considers Bertrand competition instead of Cournot competition. See Tirole's (1988) description of her model.

<sup>15</sup>GN's model must be enriched with the possibility that firms can give up R&D any time. The subgame perfect strategy is to give up R&D when the competitor is successful. This extension does not change the equilibrium of the whole game, and so GN's analysis stays valid.

## 5 Modification: Auction Setting

In this section, we show that the results change substantially if we consider a research process à la GN. Their process is commonly interpreted as a first-price auction with complete and perfect information (Reinganum 1984).

Consider the following setting:  $I$  and  $R$  bid for the service of a firm which implements a research project for the winner. This need not be taken literally. GN's interpretation is that the firms are in a race, and the firm which invests most wins and patents the innovation. Another interpretation is that firms compete for scarce and essential resources, and so only the firm which invests the most gets them. The auction is held before it is clear whether the research project will be successful.<sup>16</sup> A firm's valuation is its willingness to pay for victory, i.e., the difference in its expected profit between winning and losing.

**Proposition 3:** The incumbent always wins the auction if the innovation is non-drastic.

So with an auction setting and a non-drastic innovation,  $R$  never does research,  $I$  always does, and so there is never entry. This coincide completely with GN. Intuitively, if  $I$  wins the auction,  $R$  necessarily loses it. So preemption is always possible. If the innovation is non-drastic, then, by virtue of the efficiency effect,  $I$ 's valuation is higher than  $R$ 's, and preemption is worthwhile. In contrast, when the innovation is drastic firms' valuations are the same. Then one has to specify a tie breaking rule. However, if there is only a bit of uncertainty whether an innovation is indeed drastic,  $I$ 's valuation is higher than  $R$ 's, and so  $I$  will win the auction. So generically,  $R$  never does research, and entry never occurs.

Our analysis in this section takes us further than the analysis of GN, since we consider an uncertain research process.<sup>17</sup>

## 6 Conclusions

In our model the incumbent decides before the potential entrant about investing in research. This implies that the incumbent can discourage the potential entrant from investing by investing itself. This preemption effect is almost perfect when the success probability is high, and toothless when it is low. In GN preemption is always

<sup>16</sup>The other possibility, that the auction is held afterwards is just a special case of the former: a project with a success probability of one is auctioned. This is the case considered by GN who do not allow for uncertainty.

<sup>17</sup>GN consider uncertainty verbally, but it is not clear to us what type of uncertainty they mean. Yi (1995) couples an auction with RE's model, and his result is that the entrant will never do research, even if the innovation is drastic.

## 7 Appendix

possible, in RE never. Consequently, the efficiency effect is the driving force in GN's model, in our model when the success probability is high, and does not play any role in RE's. When preemption is not possible, the efficiency effect is not important, and the replacement effect steps in. Hence it rules in RE, in our model when the success probability is low, and not at all in GN. The probability that a monopoly persists is below one-half in RE, one in GN, and between three-quarter and one in our model. So regarding the relevant effects and the persistence of monopoly, we take a position between RE and GN.

However, there are also aspects in which our model is totally different. In RE and GN, it can – in contrast to our model – never occur that both firms yield an innovation.<sup>18</sup> GN consider no uncertainty in the research process, in RE the uncertainty concerns the arrival date of the innovation. In our model it is uncertain whether a firm's idea is realizable. Hence it is uncertain whether a firm gets an innovation, even when it invests in research.

In a previous version of the paper we consider (i) heterogenous research costs or success probabilities, (ii) patents, (iii) Cournot competition, (iv) correlated success probabilities, (v) a different timing where the potential entrant observes incumbent's success or failure in R&D before it decides about investment, and (vi) process innovations. Our results turn out to be – with small austerities – robust.

### 7 Appendix

It is useful to define some additional variables.  $\mu_J(p)$  denotes the probability that firm  $J \in \{I, R\}$  will invest in research,  $\tau(p)$  the probability that entry will occur. Obviously  $\tau(p) = p\mu_R(p)$ . Both variables are measured after  $p$  and before  $z$  is drawn.

#### 7.1 Proof of Proposition 1

(i) From Lemma 2 we get that

$$\mu_I(p) = \int_0^{(1-\pi(\bar{c},-))(1-p)p} h(z|p)dz + \int_{\pi(\underline{c},\bar{c})(1-p)p}^{\pi(\underline{c},\bar{c})p} h(z|p)dz$$

and

$$\mu_R(p) = \int_0^{(1-\pi(\bar{c},-))(1-p)p} h(z|p)dz + \int_{(1-\pi(\bar{c},-))(1-p)p}^{\pi(\underline{c},\bar{c})(1-p)p} h(z|p)dz.$$

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<sup>18</sup>This is true even if we abstract from the patent race interpretation of their models. See Section 4.6.

## 7 Appendix

Hence,  $\mu_I(p) > \mu_R(p)$  if and only if

$$\int_{\pi(\underline{c}, \bar{c})(1-p)p}^{\pi(\underline{c}, \bar{c})p} h(z|p) dz > \int_{(1-\pi(\bar{c}, -))(1-p)p}^{\pi(\underline{c}, \bar{c})(1-p)p} h(z|p) dz. \quad (6)$$

So  $\mu_I(p) > \mu_R(p)$  if and only if it is more likely that the replacement effect will determine the outcome than that the efficiency effect will.

Using A1, (6) is

$$p^2 \pi(\underline{c}, \bar{c}) > (1-p)p [\pi(\underline{c}, \bar{c}) - (1 - \pi(\bar{c}, -))].$$

So  $\mu_I(p) > \mu_R(p)$  if (and only if)  $p > \dot{p} := \frac{\pi(\underline{c}, \bar{c}) - (1 - \pi(\bar{c}, -))}{2\pi(\underline{c}, \bar{c}) - (1 - \pi(\bar{c}, -))}$ . From Lemma 1 it is clear that  $\dot{p} \in (0, 1)$ . Similarly,  $\mu_I(p) > \mu_R(p)$  if (and only if)  $p < \dot{p}$ .

(ii) A1 implies that  $h(z|p) = h(p) \forall z \in \mathcal{S}$ . By the definition of a density, it must hold that

$$\int_0^{p\pi(\underline{c}, \bar{c})} h(p) dz \leq 1$$

and hence

$$h(p) \leq \frac{1}{p\pi(\underline{c}, \bar{c})}.$$

Together with Lemma 2 this implies that

$$\mu_R(p) = h(p)\pi(\underline{c}, \bar{c})(1-p)p \leq \frac{1}{p\pi(\underline{c}, \bar{c})}\pi(\underline{c}, \bar{c})(1-p)p = 1-p.$$

Hence,  $\tau(p) = p\mu_R(p) \leq p(1-p)$ . ■

### 7.2 Proof of Proposition 2.

Start with some notation. The expected  $p$ , conditional that firm  $J$  invests in research, is

$$P_J := \int_0^1 p f_J(p) dp,$$

where

$$f_J(p) := \frac{g(p)\mu_J(p)}{\int_0^1 g(r)\mu_J(r) dr}$$

is the density of  $p$ , conditional that  $J \in \{I, R\}$  invests in research. The associated distribution function is

$$F_J(p) := \int_0^p f_J(q) dq.$$

## 7 Appendix

*Step 1.* Using Lemma 2 and A1,

$$\begin{aligned} & \frac{d\left(\frac{\mu_I(p)}{\mu_R(p)}\right)}{dp} \\ = & \frac{[\pi(\underline{c}, \bar{c}) - (1 - \pi(\bar{c}, -))] \pi(\underline{c}, \bar{c})(1 - p) + [(1 - \pi(\bar{c}, -))(1 - p) + \pi(\underline{c}, \bar{c})p] \pi(\underline{c}, \bar{c})}{[\pi(\underline{c}, \bar{c})(1 - p)]^2} \\ = & \frac{1}{(1 - p)^2}. \end{aligned}$$

Since  $\frac{f_I(p)}{f_R(p)} = \frac{\mu_I(p)}{\mu_R(p)} \frac{\int_0^1 g(r)\mu_R(r)dr}{\int_0^1 g(r)\mu_I(r)dr}$ , it follows directly that  $d\left(\frac{f_I(p)}{f_R(p)}\right)/dp > 0 \forall p \in (0, 1)$ .

*Step 2.* Claim:  $f_I(p) < f_R(p)$  for  $p \rightarrow 0$ , and  $f_I(p) > f_R(p)$  for  $p \rightarrow 1$ .

Proof: Since  $g(p)$  and  $\mu_J(p)$  are positive and finite  $\forall p \in (0, 1]$ , it follows that  $f_J(p)$  is positive and finite  $\forall p \in (0, 1]$  as well. From Step 1,  $d\left(\frac{f_I(p)}{f_R(p)}\right)/dp > 0 \forall p \in (0, 1)$ , and by definition  $\int_0^1 f_I(p)dp = 1$  and  $\int_0^1 f_R(p)dp = 1$ . Hence it must hold that  $f_I(p) < f_R(p)$  for  $p \rightarrow 0$ , and  $f_I(p) > f_R(p)$  for  $p \rightarrow 1$ .

*Step 3.* Claim: there exists a  $\tilde{p} \in (0, 1)$  such that  $f_I(p) = (>, <)f_R(p)$  for  $p = (<, >)\tilde{p}$ .

Proof: By definition,  $\frac{f_I(p)}{f_R(p)} = \frac{\mu_I(p)}{\mu_R(p)} \frac{\int_0^1 g(r)\mu_R(r)dr}{\int_0^1 g(r)\mu_I(r)dr}$ , and from Step 1,  $\frac{d\left(\frac{\mu_I(p)}{\mu_R(p)}\right)}{dp} = \frac{1}{(1-p)^2}$ . Hence,  $\frac{d\frac{f_I(p)}{f_R(p)}}{dp}$  is positive and finite  $\forall p \in (0, 1)$ . So  $\frac{f_I(p)}{f_R(p)}$  is continuous and increasing in  $p$ ,  $\forall p \in (0, 1)$ . By the intermediate value theorem and Step 2, there exists a  $\tilde{p} \in (0, 1)$  such that  $f_I(\tilde{p}) = f_R(\tilde{p})$ . Since  $\frac{f_I(p)}{f_R(p)}$  is increasing in  $p \forall p \in (0, 1)$  (Step 1), and  $f_I(p) > f_R(p)$  for  $p \rightarrow 1$  (Step 2), it holds that  $f_I(p) > (<)f_R(p)$  for  $p < (>)\tilde{p}$ .

*Step 4.* We next show that  $f_I(p)$  first order stochastically dominates  $f_R(p)$ . This is true, if and only if

$$\int_0^x f_I(p)dp \leq \int_0^x f_R(p)dp \quad \forall x \in (0, 1], \quad (7)$$

see Proposition 6.D.1 in Mas-Colell, Whinston, and Greene (1995, p. 195). We claim that this inequality is strict  $\forall x \in (0, 1)$ . Then we have almost always something like strict first order stochastic dominance.

Proof by contradiction: Suppose that there exists a  $\hat{x} \in (0, 1)$  such that  $\int_0^{\hat{x}} f_I(p)dp \geq \int_0^{\hat{x}} f_R(p)dp$ .

*Case 1:*  $\hat{x} \leq \tilde{p}$ . From Step 3,  $f_I(p) < f_R(p) \forall p \in (0, \hat{x}) \subseteq (0, \tilde{p})$ , and  $f_I(\tilde{p}) = f_R(\tilde{p})$ . Hence,  $\int_0^{\hat{x}} f_I(p)dp \geq \int_0^{\hat{x}} f_R(p)dp$  is false.

*Case 2:*  $\hat{x} > \tilde{p}$ . If  $\int_0^{\hat{x}} f_I(p)dp \geq \int_0^{\hat{x}} f_R(p)dp$ , then  $\int_{\hat{x}}^1 f_I(p)dp \leq \int_{\hat{x}}^1 f_R(p)dp$ , since  $\int_0^1 f_I(p)dp = 1$  and  $\int_0^1 f_R(p)dp = 1$ . But from Step 3,  $\forall p \in (\hat{x}, 1] \subseteq (\tilde{p}, 1]$  it is true that  $f_I(p) > f_R(p)$ , and hence  $\int_{\hat{x}}^1 f_I(p)dp \leq \int_{\hat{x}}^1 f_R(p)dp$  is false.

Note, (7) holds with equality for  $x = 1$ , since  $\int_0^1 f_I(p)dp = \int_0^1 f_R(p)dp = 1$ .

## References

Step 5. Using the definition of  $P_R$  and  $P_I$ , we get by integrating by parts that

$$P_I = 1 - \int_0^1 F_I(p)dp, \quad P_R = 1 - \int_0^1 F_R(p)dp.$$

From Step 4 we know that  $F_I(p) < F_R(p) \forall p \in (0, 1)$ , and  $F_I(1) = F_R(1)$ . Hence,  $\int_0^1 F_I(p)dp < \int_0^1 F_R(p)dp$  and so  $P_I > P_R$ . ■

### 7.3 Proof of Proposition 3

$I$ 's valuation  $v_I$  is given through the  $z$  which equates (5), and so  $v_I = p$ . Similarly, from (1) we get  $v_R = \pi(\underline{c}, \bar{c})p$ . If the innovation is non-drastic,  $\pi(\underline{c}, \bar{c}) < 1$ , and hence  $v_I > v_R \forall p \in (0, 1]$ . Since we consider a first-price auction with complete and perfect information,  $I$  will always win the auction. ■

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