Stackelberg Equilibrium with Many Leaders, U-shaped Average Costs, and Endogenous Number of Followers.

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Abstract

This paper features the equilibria of a Stackelberg model with U-shaped average costs, m leaders, and endogenous number of followers, with focus on entry preemption on the part of the leaders.

Keywords: Stackelberg competition; Free entry; Entry preemption

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1. Introduction

A recent IO literature on entry deterrence shows that in a Stackelberg model where firms face a constant marginal cost and a positive setup cost, leaders block entry of potential competitors, whenever the equilibrium number of followers is determined by a free entry condition. See, for instance, Etro (2008) and Tesoriere (2008). The reason is that, under free entry for the followers, any leader anticipates that the aggregate equilibrium output must be large enough to dissuade an additional entrant. Since marginal costs are constant, the leader prefers to produce this large output itself, *i.e.* the leaders compete to preempt entry. This paper extends this reasoning to the case of U-shaped average costs and shows that leaders prevent entry if and only if either the limit output permits positive markup or their number is sufficiently large.

2. Model

Consider a market for an homogeneous product with twice continuously differentiable, downward sloping inverse demand $p(.)$. Firms operating in this market share the same technology, embedded in the cost function

$$c(q_z) = \begin{cases} 0, \text{ if } q_z = 0 \\ F + g(q_z), \text{ if } q_z > 0 \end{cases}, \tag{1}$$

where q_z is the output level of the firm under consideration, z, $g(.)$ is a twice continuously differentiable, positive function, and $F > 0$ is a setup cost. The assumptions are made that $g'(.) > 0$, $g''(.) > 0$, and that the average cost $AC(.)$ is U-shaped, with unique minimum at the output level $\sigma > 0$. The profit function of firm z is $\pi(q_z, q_{-z}) = p(q_z + q_{-z})\, q_z - c(q_z)$, where q_{-z} is the aggregate output of firms other than z. Output levels are strategic substitutes, *i.e.* $p''q_z + p' < 0$, which, in turn, implies that $\pi(q_z, q_{-z})$ is strictly concave in own output.

This paper explores the conditions under which a group of established incumbents in the mentioned scenario forestalls entry of potential competitors. The major assumption here is that the number of such competitors actually entering the market is endogenously determined by the model. Accordingly, the issue at hand is analyzed

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through the following game. At stage one, m *leaders* simultaneously decide upon their respective output levels. Once these production decisions are made, they become known to a *large number* of potential entrants, or *followers*. At stage two, the followers simultaneously decide whether or not to enter the market. At stage three, those followers that have decided to enter the market, simultaneously choose their respective output levels. Finally, the market opens and the aggregate output is sold at the market clearing price.

A strategy for a leader i is an output level $x_i \in R$. At stage two, a follower j announces an entry decision $m_j \in M = \{0, 1\}$, where 0 stands for *not to enter* and 1 for *to enter*. Let $\mathbf{m} = (m_1, m_2, .., m_j, ..)$ denote the profile of entry decisions coming from stage two. $\mathbf{m}$ is an element of A, the set of sequences whose elements are in M. A strategy for a follower j is a pair $s_j = \{m_j, \phi_j\} \in M \times \Phi$, where $m_j \in M$, $\phi_j : A \times R \to R$ is a map assigning the production level of firm j in stage three to the profile $\mathbf{m}$ and to the aggregate output of the leaders Z, and Φ is the set of these functions. It is assumed that $\phi_j(\mathbf{m}, Z) = 0$, for any Z, if $m_j = 0$. Let $Q(Z) \geq 0$ be the aggregate output of the followers producing at stage three, when each of them, j, follows s_j, and the aggregate output of the leaders is Z. The payoff function of a leader i is

$$\Pi(x_i, Z_i, \mathbf{s}) = \pi[x_i, Z_i + Q(Z_i + x_i)],$$

where Z_i is the aggregate output of leaders other than i, and $\mathbf{s}$ is the vector of strategies played by the followers. The payoff function of a follower j is

$$\Pi(s_j, \mathbf{s}_{-j}, Z) = \pi[\phi_j(\mathbf{m}, Z), Z + Q_{-j}],$$

where Q_{-j} is the aggregate output of followers other than j.

A subgame perfect equilibrium (SPNE) of the game at hand is a strategy profile $\{\mathbf{x}^*, \mathbf{s}^*\}$ such that for any follower j the following holds:

$$\pi\left[\phi_j^*(\mathbf{m}, Z), Z + \underline{Q_{-j}^*}\right] \geq \pi\left[\phi_j(\mathbf{m}, Z), Z + \underline{Q_{-j}^*}\right], \tag{2.a}$$

for any $Z \geq 0$, any $\mathbf{m} \in A$, and any $\phi_j \in \Phi$; and

$$\pi\left[\phi_j^*(\mathbf{m}^*, Z), Z + Q_{-j}^*\right] \geq \pi\left[\phi_j^*\left(\left(m_j, \mathbf{m}_{-j}^*\right), Z\right), Z + Q_{-j}^*\right], \tag{2.b}$$

for any $Z \geq 0$ and for any $m_j \in M$; and for any leader i the following holds:

$$\pi[x_i^*, Z_i^* + Q^*(Z_i^* + x_i^*)] \geq \pi[x_i, Z_i^* + Q^*(Z_i^* + x_i)], \tag{2.c}$$

where use is made of the following definitions: $\mathbf{x}^*$ and $\mathbf{s}^* = \{\mathbf{m}^*, \boldsymbol{\phi}^*\}$ are the vectors associating to any leader i the production level x_i^* and to any follower j the entry decision $m_j^* \in \mathbf{m}^*$ and the production rule $\phi_j^* \in \boldsymbol{\phi}^*$, respectively; $\underline{Q_{-j}^*}$ and Q_{-j}^* are the aggregate output of followers other than firm j when each of them follows the strategy assigned to it by $\boldsymbol{\phi}^*$, and by $\mathbf{s}^*$, respectively; $\mathbf{m}_{-j}$ is the vector of entry decisions of followers different than j; and Z_i^* and $Q^*(Z)$ is the aggregate output of leaders different than i and of the followers when each of them is following the strategy assigned to it by $\mathbf{x}^*$ and by $\mathbf{s}^*$, respectively.

3. Equilibria

Condition $(2.a)$ above says that, given the profile of entry decisions and the aggregate output of the leaders, the equilibrium strategy ϕ_j^* of a follower j determines a best response to any aggregate output of the other firms. This means that the production decisions of the n followers that have entered the market induce an equilibrium of the n-firm Cournot game with the same technology as in the present scenario and demand $p\,(.+Z)$. It is well known that for the setting here, there exists a unique symmetric such equilibrium, where each follower produces the same quantity $q^n\,(Z)$.[1] The associated per firm profit is strictly decreasing whether in Z and in n.

Condition $(2.b)$ says that for any aggregate output of the leaders, the entry decisions of the followers are conditional on the subsequent equilibrium in stage three. The analysis here disregards the integer constraint and treats the number of followers $n\,(Z)$ active at stage three as a real number. This approach is adopted for analytical easy, and is briefly discussed in section 4. Under the assumption that the per firm Cournot equilibrium revenue tends to zero as n grows without bound, $n\,(Z)$ is uniquely determined for any Z and satisfies $\pi\left[\phi_j^*\left(\mathbf{m}^*, Z\right), Z + \left(n\,(Z) - 1\right)\phi_j^*\left(\mathbf{m}^*, Z\right)\right] = 0$, while $\phi_j^*\left(\mathbf{m}^*, Z\right)$ satisfies the first order condition for an interior, symmetric equilibrium $\partial\pi\left[\phi_j^*\left(\mathbf{m}^*, Z\right), Z + \left(n\,(Z) - 1\right)\phi_j^*\left(\mathbf{m}^*, Z\right)\right]/\partial q_j = 0$, for any j choosing $m_j = 1$. Following Etro, totally differentiating the previous two conditions, one obtains $d\phi_j^*\left(\mathbf{m}^*, Z\right)/dZ = 0$, $dn\,(Z)/dZ = -1/\phi_j^*\left(\mathbf{m}^*, Z\right)$, and $d\phi_j^*\left(\mathbf{m}^*, Z\right)/dn = 0$ which, in turn, yield $\partial Q^*\left(Z_i + x_i\right)/\partial x_i = -1$, for any leader i. All the above means that as long as there are some followers producing a strictly positive output at stage three, total SPNE output l does not depend on Z. It is assumed that entry is not blockaded, *i.e.* that the monopoly output is smaller than l. Let now Y satisfy $\pi\left(q\,(Y), Y\right) = 0$, where $q\,(.)$ denotes the unique root of $\partial\pi\left(q_i, .\right)/\partial q_i = 0$. Note that $q\,(.)$ is decreasing, while $(.+q\,(.))$ is increasing. Thus Y is unique, and satisfies $n\,(Y) = 1$. It follows that $l = Y + q\,(Y)$.

Finally, condition $(2.c)$ says that the equilibrium strategy of a leader i specifies a best response to the aggregate output of the other leaders, conditional on the equilibrium strategy of the followers. Unlike $n\,(Z)$, the number of leaders m is given and will be treated as an integer. In addition, the assumption is made that a leader produces a positive output only if it anticipates strictly positive equilibrium profits. So, the best response of a leader i to an aggregate output $Z_i \geq Y$ is zero.

Assume, for now, that $p\,(l) \geq g'\,(l)$. It first follows that for any Z_i satisfying $Y > Z_i \geq 0$, and any x_i satisfying $l - Z_i > x_i \geq q\,(Y)$,

$$\Pi\left(l - Z_i, Z_i, \mathbf{s}^*\right) - \Pi\left(x_i, Z_i, \mathbf{s}^*\right) = p\,(l)\left(l - Z_i - x_i\right) - c\left(l - Z_i\right) + c\left(x_i\right) =$$

$$p\,(l)\left(l - Z_i - x_i\right) - \int_{x_i}^{l-Z_i} g'\,(u)\,du > \left(p\,(l) - g'\left(l - Z_i\right)\right)\left(l - Z_i - x_i\right) > 0. \quad (3)$$

Inequality (3) implies that for $Z_i < Y$, any $x_i < l - Z_i$ is a strictly dominated action. Moreover, since $\Pi\left(q\,(Y), Z_i, \mathbf{s}^*\right) > 0$, through producing $l - Z_i$, the leader obtains strictly positive profit. Further, the *interior Cournot reaction function* never mandates to produce more, *i.e.* $q\,(Z_i) < l - Z_i$, because $(.+q\,(.))$ is strictly increasing. Thus, the reaction function of a leader i is given by

$$r\,(Z_i) = \begin{cases} 0 \text{ if } Z_i \geq Y \\ l - Z_i \text{ if } Z_i < Y \end{cases}. \quad (4)$$

[1] Moreover, there are not asymmetric equilibria where two firms produce strictly positive but different output levels. This study focuses on the symmetric equilibrium only.

The most important effect of (4) is that the aggregate equilibrium output of the leaders is equal to l. Hence, at stage two the followers remains out of the market.[2] In equilibrium, each leader i produces the equilibrium output $x_i^m = l - \sum_{z \neq i} x_z^m$, with

$$x_i^m > 0 \text{ if } \sum_{z \neq i} x_z^m < Y \text{ and } x_i^m = 0 \text{ if } \sum_{z \neq i} x_z^m \geq Y. \tag{5}$$

There are infinitely many such equilibria. In particular, any configuration where $k \leq m$ leaders produce the aggregate output l, $1 \leq k < \nu$, condition (5) is satisfied, and the remaining $m - k$ leaders produce zero each, is an equilibrium, where $\nu = l/q(Y)$. As reported, at each of these configurations no follower enters the market, *i.e.* entry is preempted. To see that $k < \nu$, notice that otherwise at least one leader would produce an output level not larger than $q(Y)$ and hence obtain non positive profit. Finally, any leader i producing $x_i^m > 0$ makes strictly positive profits, the latter due to $x_i^m > l - Y = q(Y)$. Given (4) and the definition of SPNE, there are not other equilibria. The findings above are similar to those obtained by the mentioned literature for the case of constant marginal costs.

Assume now that $p(l) < g'(l)$. Since $p(l) > g'(q(Y))$, there exists an output level $\omega > q(Y)$, such that $p(l) = g'(\omega)$. Now, as already noted, for $Z_i < Y$, $Z_i + q(Z_i) < l$. By strict concavity of the profit function in own output, this means that for any $x_i > l - Z_i$, marginal revenue for a leader i is smaller than marginal cost. In particular, $\Pi(x_i, Z_i, \mathbf{s}^*)$ is increasing in x_i for $x_i < \min\{\omega, l - Z_i\}$, while it is decreasing for larger output levels. The latter means that the reaction function of i is

$$r(Z_i) = \begin{cases} 0 \text{ if } Z_i \geq Y \\ \min\{\omega, l - Z_i\} \text{ if } Z_i < Y \end{cases}. \tag{6}$$

A major consequence of (6) is that the aggregate equilibrium output cannot exceed l. This, along with free entry at stage two, means that total equilibrium output is just l. As ω is a constant, positive term, there exists an integer number k_1 such that $m\omega > Y$ if and only if $m \geq k_1$. In addition let k_2 satisfy $m\omega > l$ if and only if $m \geq k_2$. Note that either $k_2 = k_1$ or $k_2 = k_1 + 1$. A first point is that, if $m < k_1$, the configuration where the leaders produce ω each and $n(m\omega)$ followers enter the market and produce the aggregate output $l - m\omega$ is an equilibrium. Since $\omega < l$, this would be the case if $m = 1$. So, unlike the case $p(l) > g'(l)$, one leader only does not prevent entry. If $k_2 \neq k_1$, then the last mentioned configuration is sustainable as equilibrium for $m = k_1$ too. If $m \geq k_2$, then any configuration with $\mu \leq m$ leaders producing the aggregate output l, with $k_1 \leq \mu < \nu$, and with the equilibrium output x_i^m of each leader i satisfying $x_i^m = \min\{\omega, l - Z_i\}$, $x_i^m > q(Y)$, the remaining $m - \mu$ leaders producing zero, and no followers entering the market is an equilibrium. The fact that $\mu < \nu$ follows along the same line as in the previous case. Finally, each leader i producing $x_i^m > 0$ obtains strictly positive profits, for the reason that the average cost curve is strictly U-shaped, that, in turn, implies $p(l) > AC(x)$ for any x such that $\omega \geq x > q(Y)$. As in the previous case, there are not other equilibria, due to (6) and the fact that the argument above applies to any number m of leaders.[3]

The following proposition summarizes the results above.

[2] More exactly, $Q^*(l) = 0$.

[3] Etro provides an example with one leader and quadratic costs in the same vein as the present analysis.

Proposition. (a) *Assume* $p(l) \geq g'(l)$. *If* $m = 1$, *then there is a unique SPNE where the only leader produces* l *and no follower enters the market. If* $m > 1$, *then there are infinitely many SPNE at each of which* $k \leq m$ *leaders produce the aggregate output* l, *with* $1 \leq k \leq \nu$, *condition* (5) *is satisfied, the remaining* $m - k$ *leaders produce zero each, and no followers enter the market.* (b) *Assume* $p(l) < g'(l)$. *If* $m < k_1$, *then there is a unique SPNE where each leader produces* ω *and* $n(m\omega)$ *followers enter the market and produce the aggregate output* $l - m\omega$. *If* $k_2 \neq k_1$, *the latter is the unique equilibrium for the case* $m = k_1$ *as well. If* $m \geq k_2$, *then there are infinitely many equilibria at each of which* $\mu \leq m$ *leaders produce the aggregate output* l, *with* $k_1 \leq \mu < \nu$, *the equilibrium output* x_i^m *of each leader satisfies* $x_i^m = \min\{\omega, l - Z_i\}$, $x_i^m > q(Y)$, *the remaining* $m - \mu$ *leaders produce zero each, and no follower enters the market. There are not other SPNE.*

4. Concluding remark

Part (a) of the proposition remains valid also when considering the number of followers as an integer, provided l is substituted with Y, which is the usual reference for the limit output in the IO literature. The use of l in this paper is motivated by the choice made here of treating n as a real number, which, in turn, greatly simplifies the analysis. In fact, the alternative of imposing $n(Z) \geq 1$ together with a zero profit condition for the followers would leave the limit output undetermined. A possible interpretation of this setting would rephrase the technological framework through assuming that followers are small compared to leaders, in a spirit close to Novshek (1980). In this case, the cost function of a follower would be $c_\alpha(q) = \alpha c(q/\alpha)$, where α is the size of the follower and $\alpha\sigma$ is the follower's minimum efficient scale. The idea is that for *small* α, no output level $Z < l$ would preempt entry, since a follower best reacting to Z would obtain a profit not smaller than $p(Z + \alpha\sigma)\alpha\sigma - c_\alpha(\alpha\sigma) > p(l)\alpha\sigma - c_\alpha(\alpha\sigma) > (AC(\sigma) - AC_\alpha(\alpha\sigma))\alpha\sigma = 0$.

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