Externality and Timing of Entry in Oligopoly Models

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Abstract

Using models of imperfect competition, this paper analyzes endogenous timing of entry with the possibility that firms learn from the actions of others. Under sequentially entry, the late-moving firm pays a lower entry cost due to positive externalities that arise from the first firm's entry. In the case of two symmetric profit-maximizing private firms, we find that multiple equilibria which stem from the trade-off between entering early or late. When the analysis is carried out with a welfare-maximizing public firm and a profit-maximizing private firm, we show that equilibrium outcome depends on the size of cost inefficiency parameter of the public firm. Finally, we also conduct an analysis where the public firm acts as a strategic initiator of the costly initial expenditure before leaving the market to the private firms.

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1 Introduction

In many situations, firms learn from the actions of others. R&D program, multinational enterprises, exploration for oil and gas reserves, to mention a few, serve as an example of learning game. In the case of R&D program, one firm's decision to invest early can confer positive externalities to other firms working on similar problems. For instance, Netscape first entered in the web browser market and established its leading position in the market. Netscape's main rival, Microsoft, entered the browser market at a later point. With better knowledge and lower cost of technological adoption, Microsoft was able to compete successfully against Netscape. As the history shows, Microsoft could not be prevented from entry by any preemptive strategy by Netscape.[1] Another example is multinational firms contemplating entry into foreign market. Thus, once foreign investment by one firm is made, it makes it easier for others to follow suit. Aitken, Hanson and Harrison (1997) provide a good example of how the entry by one Korean garment exporter in Bangladesh attracted hundreds of exporting enterprises which essentially led to the development of garment export industry in Bangladesh. Finally, using oil lease auctions data for U.S. firms between 1954 and 1969, Hendricks, Porter and Boudreau (1987) document that twenty-seven percent of all the leased tracts were allowed to expire without any wells being drilled.

In situations like these, there are incentives to wait as firm can learn from its predecessor who initiates the costly initial investment. The existence of such externality may lead to market failure in the form of inefficient underinvestment (e.g., Pindyck, 2005). Unlike the real option models (e.g., Dixit and Pindyck (1994)), where firms usually learn about market conditions simply by waiting, the present analysis is based on situations where follower learns learns instead from the actions of the leader.

In this paper we develop a multi-period duopoly game, in which entry patterns are determined endogenously. We first describe the baseline model which consists of two profit-maximizing firms. In order to model the role of learning, we assume that each firm is subject to an entry cost which depends on the number of firms that had already entered in the previous period. Thus, if two firms enter simultaneously in a particular period, both incur similar entry costs and consequently no learning takes place from the competition. By contrast, if one firm enters early, it generates positive externalities for

[1] This example is adapted from Brown and Chiang (2003).

the late-comers since the latter firm pays a lower entry cost. However, although the first entrant pays a higher entry cost it also enjoys monopoly profit until the second firm enters in the market. While the second entrant forgoes monopoly profit but benefits from lower entry cost.[2] This trade-off between entering early or late can introduce the possibility of multiple equilibria in which firms enter in different periods. Multiple equilibria are quite common in entry models.[3] One advantage of our analysis is that the multiple equilibria can be ranked in terms of aggregate welfare.

Next we extend the baseline model to the case where firms are asymmetric both in terms of their marginal costs[4] of production, as well as their objective functions. In particular, we analyze the entry game between a public and a private firm.[5] In many countries, public firms compete with private firms in imperfectly competitive industries such as airline, banking, electric power, health care and telecommunication. The idea is to investigate to what extent a public firm's presence affects the equilibrium entry patterns where initial entry generates positive externalities for other potential entrants. Unlike profit-maximizing private firms, public firms maximize social surplus. Throughout the paper, we retain the assumption that public firm has a cost disadvantage vis-á-vis private firm. It is shown that equilibrium entry patterns depend on the degree of cost inefficiency of the public firm. Thus, when it is assumed that the public firm is relatively less inefficient, entry does not take place and public firm remains the sole producer in the market. The result is overturned when the level of inefficiency is set sufficiently high, in which case public firm does not produce while only the private firm operates in the market. We also derive conditions for the private firm's decision to enter either in first or second period, which is related to moderate level of inefficiency of the public firm.

In the final extension of our baseline model, we analyze a situation in which the public firm makes the costly initial investment and then leave the market to the private firms for production. Basically, we want to examine the implications of this type of behavior by public firm on the endogeneity of equilibria, as well as social welfare. We find that public

[2]Note that, entry timing needs not be taken place in successive periods. Thus, if one firm enters in the first period, the other firm may either enter in the second period or delays its entry to a later period. However, prolonging the option to delay will only result in foregone profits and not lowering the entry cost further.

[3]See, for instance, Lin and Saggi (2002), Brown and Chiang (2003), and Pakes (2004).

[4]Lin and Saggi (2002) consider this scenario with two private firms.

[5]This kind of market structure is famously known as mixed oligopoly in the Industrial Organization literature.

firm's role as the strategic initiator of the investment provides an important element to equilibrium entry patterns.

The plan of this paper is as follows. Section 2 provides a brief review of the literature. Section 3 presents the baseline model of pure duopoly. Section 4 offers a linear demand illustration of the baseline model and derives the equilibrium entry conditions parametrically. Sections 5 analyzes the mixed duopoly version of the game. Section 6 extends the baseline model in which public firm behaves as the strategic initiator of the initial investment. Section 7 briefly concludes by presenting areas of further research. Proofs are relegated to an Appendix.

2 Related literature

Our paper is related to two different strands of literature. First, the literature discusses the endogenous entry patterns, assuming pure oligopoly models.[6] Recent contribution includes Lin and Saggi (2002), Quint and Einav (2005), and Pindyck (2005). A second strand of the literature investigates equilibrium entry pattern in mixed oligopoly models. Work along this line includes Ware (1986), Nett (1990) and Fershtman (1990). We proceed in two steps. First, we provide a short overview of the literature based on pure oligopoly models. Then we briefly review the relevant models proposed in the mixed oligopoly literature.

Lin and Saggi (2002) developed a two-period duopoly model in which the cost of entry depends upon the relative entry times of the two firms. Thus, following an entry by first firm the longer the second entrant delays its entry, the more it learn about the market conditions, and the lower its entry cost. Of course, more delayed entry also means foregone profits. This trade-off between a lower entry cost and foregone profits govern the equilibrium outcomes of the model which could either be simultaneous or sequential. Thus, if firms are symmetric and when foregone profits are large relative to the benefit of lower entry cost, the model generates simultaneous entry as the unique equilibrium. When the converse is true, the model admits a sequential entry equilibrium where one of the firms enters after an optimal waiting period endogenously determined in the model.

[6]Our analysis is also related to the social learning literature in which firms use signal structure to decide on the relative timing on entry into a market. See Chamley (2004) for an overview of the field. To keep our analysis short and to the point, we refrain from comparing our results to theirs.

On the other hand, if firms differ in their marginal costs of production, multiple equilibria result. In the asymmetric case, although the socially desirable outcomes dictates that the low-cost firm enters first, in equilibrium it is equally likely that either of them may enter first. Moreover, it is not clear whether social welfare is higher when the low-cost firm enters first or second.

It is also useful to relate our paper to Appelbaum and Weber (1994), Maggi (1996), Brown and Chiang (2003), and Quint and Einav (2005), who also studied endogenous entry in pure oligopoly. Appelbaum and Weber (1994) show that equilibrium outcomes are always symmetric in which firms simultaneously enter either before or after the resolution of uncertainty.[7] By contrast, Maggi (1996) shows that even with identical players there may be asymmetric equilibria where one firm commits to early investment, while the other firm follows a wait-and-see strategy. Using a symmetric duopoly game under demand uncertainty, Brown and Chiang (2003) show that multiple equilibria exist including two Cournot-type equilibria and two Stackelberg-type equilibria. More recently, Quint and Einav (2005) propose a dynamic entry game in which entry costs become sunk gradually. In equilibrium, only the most profitable firms enter while the remaining firms dropout from the market.

This paper belongs also to the literature which investigates equilibrium entry pattern in mixed oligopoly models. Ware (1986) studied a two-stage, two-player entry game in which the public firm was allowed to act as an incumbent and as well as an entrant. Thus, when the public firm is assumed to be the incumbent firm, it was shown that private firm does not enter because of incumbent's commitment to the maximization of social welfare but not profit.[8] By contrast, when the public firm is contemplating entry into an industry served by a private monopoly, the latter is able successfully to deter entry by a public firm.

Fershtman (1990) considers an entry deterrence game between a partly nationalized firm[9] and a private firm. He finds that a public incumbent has more power in entry deterrence than a private incumbent. These results crucially depend on the assumption

[7]Sequential entry never emerges as an equilibrium outcome in their model. This is because of the assumptions that firms are symmetric and they do not receive any private information. See the references in Appelbaum and Weber (1994) for some early references of the literature.

[8]This trivial result is due to the assumption of identical cost structure between public and private firm.

[9]Unlike a welfare-maximizing pure public firm, a partly nationalized firm maximizes a weighted sum of its own profit and social welfare.

of constant per unit costs of output and the capacity between firms. However, when sufficient cost asymmetry is considered in favor of the private firm, he demonstrated that market entry by a private firm is possible. Contrary to Fershtman, Nett (1990) argued that the possibility of market deterrence is determined by the market conditions (demand and technology) and not by the firms' objective functions.

Recently, Matsumura and Kanda (2005) studied a quantity-setting mixed oligopoly game in which a public firm competes with many private firms. The authors allow free entry of private firms and show that in equilibrium mixed markets generate higher social benefit than pure markets involving no public firm. A limitation of the entry deterrence literature on mixed oligopoly, including the one cited above, is that the role of learning is generally ignored.

3 Private duopoly: The baseline model of entry

Consider a market is being served by two profit-maximizing firms. The industry inverse demand curve is $p_t = f(q_t)$, where $q_t = \sum_{i=1}^{2} q_{it}$ denotes industry output; and p_t is the demand price. The firms have identical cost functions $c(q_{it}) = cq_{it}$, for $i \in (1, 2)$. The game is played for infinite periods. Entry is subject to a fixed cost $F(\cdot)$, which depends upon n, the number of firms that had entered previously. Thus, if no firm had entered before the fixed cost of entry is $F(0)$; and if one firm had entered previously, the fixed cost becomes $F(1)$ such that $F(1) < F(0)$. In other words, the late-mover has an advantage.

Throughout this paper, it is assumed that there is no entry deterrence.[10] In addition, we also rule out any possibilities of collusion which can easily be justified through the presence of antitrust laws as well. To focus on the intrinsic advantage and disadvantage of timing of entry, we assume that both firms face the same cost, demand and informational conditions.

The game proceeds as follows. Each firm decides to enter in each period. There can be two situations in each period: duopoly and monopoly. We assume that when both firms

[10] To see this, consider the case of entry deterrence where the incumbent produces an output level $\overline{q}$ such that the entrant finds it unprofitable to enter. However, since the incumbent incurs a higher fixed cost (i.e., $F(0)$), the choice of $\overline{q}$ will lead to negative profit for the incumbent firm. Thus, $\overline{q}$ cannot be sustained as an entry deterrence level of output. As a result, the incumbent will revert back to some other output level (strictly less than $\overline{q}$), so that entry becomes profitable. On the other hand, for the potential rival firm entry delivers a duopoly rent, since it is the incumbents best response to accommodate to the new duopoly situation.

enter simultaneously, they play a Cournot duopoly game where output is determined non-cooperatively.

Denote by π_m and π_d the monopoly and duopoly[11] profits in each period, respectively. Firms enter only if the discounted profit over time exceeds the fixed cost where both profit and cost depend upon when the other firm enters. If they both enter at the same time, their discounted profit is $\frac{\pi_d}{1-\delta} - F(0)$. If one firm enters first and the other later, the profits respectively are, $\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)$ and $\delta\frac{\pi_d}{1-\delta} - F(1)$, where δ is the discount rate such that $0 < \delta < 1$.

In view of the discussion above, it is clear that there are four possible equilibrium outcomes of interest which need to be considered: (i) firms enter simultaneously, (ii) firms enter sequentially, (iii) only one firm enters and (iv) no firms enter. Let E_1, E_2, E_3, and E_4, denote the equilibrium outcome associated with (i), (ii), (iii) and (iv), respectively. Each of the four outcomes is now considered in detail.

3.1 Simultaneous entry (E_1)

If both firms enter in period t, it must be true that $\delta^t[\frac{\pi_d}{1-\delta} - F(0)] > \delta^{t+1}[\frac{\pi_d}{1-\delta} - F(1)] \geq 0$. In other words, moving early is more beneficial than entering late. We get the following lemma[12]

Lemma 1. *Both firms can never enter simultaneously in period $t > 1$.*

The intuition for this result is clear. If one firm enters early, the other follow the suit since the benefit of delaying entry is smaller than the instantaneous duopoly profits. Moreover, it is not feasible for firms to enter simultaneously at a later period since any one firm can always enter early and enjoy the monopoly profits. This temptation motivates both firms to consider entry in period 1.

3.2 Sequential entry (E_2)

One firm enters in period t and the other in period $(t+1)$. One would delay its entry as long as $\delta^t[\frac{\pi_d}{1-\delta} - F(0)] < \delta^{t+1}[\frac{\pi_d}{1-\delta} - F(1)]$. In other words, it pays to delay the entry by one more period since there is a greater benefit from being a late mover. If one firm

[11] We assume the demand and cost functions are to be such that there is only a unique equilibrium in the Cournot game.

[12] Lin and Saggi (2002) obtained a similar result.

delays, the other firm may choose to delay as well and enter even later or may switch to enter at an early period. Suppose, in period 1, firm 1 decides to delay its entry until period 2.

We also need to satisfy two other conditions. One is entering in second period is profitable for the late-comer or $\delta^{t+1}[\frac{\pi_d}{1-\delta} - F(1)] \geq 0$. This condition can be rewritten as $\frac{\pi_d}{1-\delta} - F(1) \geq 0$. Also, the pioneer (first entrant) finds it profitable to enter in one period ahead or $\delta^t \left[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)\right] \geq 0$. This also can be mentioned as $\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0) \geq 0$.

Now, will firm 2 subsequently enter in period 1 or decide to enter even later such as in period 3? The payoff from entering in period 1 is $[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)]$ and the payoff from entering in period 3 is $\delta^2[\frac{\pi_d}{1-\delta} - F(1)]$. Since, each firm knows that the other will delay and their profits will be discounted in subsequent periods, they will not postpone the entry further than period 2.

Thus we have established the following proposition.

Proposition 1. *(i) If* $\delta \in [0, w)$ *and* $\frac{\pi_d}{1-\delta} - F(0) > 0$ *both firms enter in the first period; (ii) If* $[\frac{\pi_d}{1-\delta} - F(0)] < \delta[\frac{\pi_d}{1-\delta} - F(1)]$, $\left[\frac{\pi_d}{1-\delta} - F(1)\right] \geq 0$ *and* $\left[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)\right] \geq 0$, *if* $\delta \in [w, 1)$, *the firms enter in period 1 and in period 2.*

where $w = \frac{\pi_m + \frac{\pi_d}{1-\delta} - F(0)}{\frac{\pi_d}{1-\delta} - F(1)}$. Proposition 1(i) states that if the advantage of being a late-comer is low or the difference $|F(0) - F(1)|$ is not too high, both firms prefer to enter during the first period. Proposition 1(ii) states that the other firm tend to delay and enter later since $F(1)$ is sufficiently low which will motivate one to postpone the entry after someone else have initiated the productions process. Lin and Saggi (2002) also obtain these equilibria with two symmetric profit-maximizing private firms.

3.3 Single entry (E_3)

One firm enters and the other does not enter if $[\frac{\pi_m}{1-\delta} - F(0)] \geq 0$ but $[\frac{\pi_d}{1-\delta} - F(1)] < 0$. In other words, there is room for only one firm to enter. If the other firm enters, both start making losses, so only one firm ceases to exist. This is possible if the demand is too inelastic and $|F(0) - F(1)|$ is too large.[13]

[13] In a complete information Cournot model, Amir and Lambson (2000) also point to the existence of this type of equilibria in which one firm or a subset of firms produce a positive quantity whereas the others are inactive, producing nothing.

3.4 No entry (E_4)

There are two possible cases of no-entry.[14] In the first case, $[\frac{\pi_m}{1-\delta} - F(0)] > 0$ but $[\frac{\pi_m}{1-\delta} - F(0)] > 0 > [\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)]$. Basically, there is enough profit for the monopoly to survive but as soon as the initial investment is made, the other firm will enter in the second period as a late-comer. This second entry will lead to duopoly from period 2 onwards. In other words, it leads to lower profit that does not make the entry of the first period profitable. Note that, second period entry is profitable if $[\frac{\pi_d}{1-\delta} - F(1)] > 0$.

In the second scenario, assume $[\frac{\pi_m}{1-\delta} - F(0)] < 0$. In other words, even if the firm is a monopoly all the way through, the profit is never going to be enough to cover the initial cost. In this case, it is the demand condition that is the sole determinant in making the no-entry decision. Formally,

Proposition 2. *(i) If* $[\frac{\pi_m}{1-\delta} - F(0)] \geq 0$ *and* $[\frac{\pi_d}{1-\delta} - F(1)] < 0$*, there is only one entry; (ii) If* $[\frac{\pi_m}{1-\delta} - F(0)] \geq 0$*,* $[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)] < 0$ *but* $[\frac{\pi_d}{1-\delta} - F(1)] > 0$ *there is no entry at all; and (iii) If* $[\frac{\pi_m}{1-\delta} - F(0)] < 0$*, there is no entry as well.*

Proposition 2(i) states that if cost reduction is not too high and there is a greater divergence between monopoly profit and duopoly one, then only single entry will prevail. This will happen if $\frac{\pi_m}{1-\delta} - \frac{\pi_d}{1-\delta}$ is large but $|F(0) - F(1)|$ is small. On the contrary, Proposition 2(ii) states that if $\frac{\pi_m}{1-\delta} - \frac{\pi_d}{1-\delta}$ is large or competition does dissipate profit to a great extent but $|F(0) - F(1)|$ is large or the late-comer has a distinct advantage, then the fear of entry of the other firm will dissuade anyone to enter at all. On the other hand, part (iii) of Proposition 2 states that the demand condition of the market is simply not big enough to have a room for even one firm.

4 A linear demand illustration

In order to characterize the model in explicit forms in terms of the parameters of the cost and demand functions, in this section we consider a simple example in which both demand and cost functions are linear:

$$p(q) = a - q_1 - q_2, \;\; a > 0$$

[14] Pindyck (2005) recently analyzes a similar game using the Prisoner's dilemma where he demonstrates that when externality in the form of learning from others is present, equilibrium outcome is the one where nobody invests.

$$c(q) = cq_i, \;\; c > 0, \;\; a > c$$

The profit function of firm i is given by

$$\pi_i = pq_i - cq_i, \quad i = 1, 2$$

The profit function π_i can be used to solve for π_d (in case of duopoly competition) and π_m (in case of monopoly outcome). Straightforward calculations show that the reduced-form profit functions, for each firm, are

$$\pi_m = (a-c)^2/4, \qquad \pi_d = (a-c)^2/9.$$

Proposition 1(i) states that for equilibrium E_1 to occur we need $\frac{\pi_d}{1-\delta} - F(0) \geq 0$. Substituting π_d and rearranging gives us **first equilibrium condition** in terms of the exogenous parameters of the model:

$$E_1: \quad (a-c) \;\; \geq \;\; 3\sqrt{(1-\delta)F(0)} \tag{1}$$

For equilibrium E_2 to occur, three conditions must be satisfied. These are (see Proposition 1(ii)): (a) $[\frac{\pi_d}{1-\delta} - F(0)] < \delta[\frac{\pi_d}{1-\delta} - F(1)]$, (b) $\frac{\pi_d}{1-\delta} - F(1) \geq 0$, and (c) $\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0) \geq 0$. Substituting π_m, π_d and rearranging, respectively, for each condition, yield

$$\begin{aligned} (a-c) &\;<\; 3\sqrt{F(0) - \delta F(1)} \\ (a-c) &\;\geq\; 3\sqrt{(1-\delta)F(1)} \\ (a-c) &\;\geq\; 6\sqrt{\frac{(1-\delta)F(0)}{9-5\delta}} \end{aligned}$$

which collectively yields the **second equilibrium condition**:

$$E_2: \quad \max\left[3\sqrt{(1-\delta)F(1)},\; 6\sqrt{\frac{(1-\delta)F(0)}{9-5\delta}}\right] \leq (a-c) < 3\sqrt{F(0)-\delta F(1)} \tag{2}$$

Next, Proposition 2(i) refers that equilibrium E_3 requires $[\frac{\pi_m}{1-\delta} - F(0)] \geq 0$ and $[\frac{\pi_d}{1-\delta} - F(1)] < 0$. Substitution and rearranging gives the **third equilibrium condition**:

$$E_3: \quad 2\sqrt{(1-\delta)F(0)} \leq (a-c) < 3\sqrt{(1-\delta)F(1)} \tag{3}$$

Finally, Proposition 2(ii-iii) state the conditions for equilibrium E_4. Even if $[\frac{\pi_m}{1-\delta} - F(0)] > 0$, but $[\frac{\pi_m}{1-\delta} - F(0)] > 0 > [\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)]$, or equivalently, $3\sqrt{(1-\delta)F(1)} \leq (a-c) < 6\sqrt{\frac{(1-\delta)F(0)}{9-5\delta}}$, no firms will enter. In addition, if $[\frac{\pi_m}{1-\delta} - F(0)] < 0$, or equivalently,

$(a-c) < 2\sqrt{(1-\delta)F(0)}$, no entry will take place as well. This leads us to obtain the **fourth equilibrium conditions**:

$$E_4: \quad (a-c) \leq 2\sqrt{(1-\delta)F(0)} \quad \text{and}$$
$$3\sqrt{(1-\delta)F(1)} \leq (a-c) < 6\sqrt{\frac{(1-\delta)F(0)}{9-5\delta}} \tag{4}$$

To summarize, equations (1) to (4) define the steady state equilibrium of the model in terms of the structural variables and fixed costs of entry. An important objective of this paper has been to understand how changes in the sunk entry cost would alter the equilibrium outcomes of the model. See Figure 1 for a graphical illustration of the equilibrium patterns of this game.

{*insert Figure 1 here*}

4.1 Welfare analysis

Here we evaluate the implications of multiple equilibria on social welfare. Social welfare is given by the sum of producers' and consumers' surplus, i.e., $W_j = \pi_j + CS_j$ indexed by $j = 1, 2, 3, 4$ denoting equilibrium outcome.[15] Using the derivations shown above it is straightforward to show that the welfare under each equilibrium has the following expression:

$$W_1 = \frac{4(a-c)^2}{9(1-\delta)} - 2F(0) \tag{5}$$
$$W_2 = \frac{3(a-c)^2}{8} + \frac{4\delta(a-c)^2}{9(1-\delta)} - F(0) - \delta F(1) \tag{6}$$
$$W_3 = \frac{3(a-c)^2}{8(1-\delta)} - F(0) \tag{7}$$
$$W_4 = 0 \tag{8}$$

We can directly obtain the following results:

Corollary 1. $W_2 > W_3 > W_1 > W_4$.

As expected, the maximum social surplus is achieved under sequential entry. Basically, the late entrant benefits from lower entry cost due to entry by the pioneer in period 1. Interestingly, social welfare is lower under simultaneous than that of single entry. The

[15] The welfare function has a subscript since profit and consumer surplus differ across equilibria.

intuition for this result is that under simultaneous entry society does not benefit from learning and incur very high entry costs, while with single entry even through learning effect is absent but the aggregate entry costs are relatively lower. Finally, under no-entry equilibrium, social welfare is affected as a result of the inefficient underinvestment by firms.

Is there a way to coordinate the investment decision of the various firms such that it benefits the society? To explore this issue, we next analyze the entry game to the context of a mixed duopoly market where competition takes place between a public and a private firm. We are interested to see how the pattern of entry will differ in the presence of a public firm. This is analyzed in the next section.

5 Entry under mixed duopoly

Consider a mixed duopoly market with one public and one private firm indexed by firm 1 and 2, respectively. To keep this section short and to the point, we analyze the game for the case of linear demand model given as $p = a - q_1 - q_2$, where p is the market price and q_i for $i = 1, 2$ represents output. The marginal costs of production, are, respectively, c and kc for private and public firm. In other word, public firm is assumed to be less efficient than the private firm with $k > 1$ denotes the inefficiency parameter.[16] The public firm's objective is to maximize welfare defined as the sum of consumer surplus and profit of all firms, whereas private firm's objective is to maximize its own profit. As before, entry is subject to a fixed sunk cost $F(\cdot)$ which depends on the number of firms n that have already entered. See Figure 2 for a graphical illustration of this game.

{*insert Figure 2 here*}

The game proceeds as follows. It is assumed that the public firm always enters in period 1, while the private firm can either enter in period 1 or 2. Thus, if the private firm simultaneously enters with public firm in period 1, no positive externalities take place and each pays $F(0)$ as the initial cost of entry. By contrast, if the private enters in period 2, it pays a lower entry cost $F(1)$ since the public firm has already entered in period 1 and paid $F(0)$. Note that, public firm's decision to participate actively in the market depends

[16] Note that, if $k = 1$ then the entire market is served by the public firm only. To avoid this trivial outcome we assume $k > 1$. See Nett (1993) for further discussion behind this assumption.

largely on the level of her cost inefficiency (i.e., the parameter k). As will be shown below, situation may arise that will cause the public firm not to produce any output letting the market to be served by the private firm only.

5.1 Private firm does not enter

In this case the entire market is served by the public firm alone, thereby yielding a monopoly outcome. Denote by N as no-entry, the profit function for firm 1 is $\pi_{1N} = (a - q_1 - kc)q_1$. The per-period social welfare function under no-entry is: $W_N = q_1(a - q_1/2 - kc) - F(0)$. Maximizing W_N and solving for output yields $q_{1N} = a - kc$.

Lemma 2. *It is never optimal to produce beyond q_{1N}.*

to see the intuition behind Lemma 2, suppose at q_{1N} no firm enters. Thus, producing beyond q_{1N} will lead to welfare loss of the amount $(1/2)(q_0 - q_{1N})^2$ in the form of deadweight loss, where q_0 is any other output which is higher than q_{1N}. Moreover, had there been entry by the private firm, it must be that her total discounted profit is greater than or equal to the fixed cost (whether it is $F(0)$ or it is $F(1)$). In that case, the change in social welfare is greater than firm 2's accumulated profit over time since private firm can only expropriate a part of the consumer surplus but not the whole of it. It follows that change in welfare must be greater than the fixed cost incurred by the private firm upon entry. Thus, entry by the private firm is worthwhile as far as the social welfare is concerned. This confirms that producing beyond the stipulated amount q_{1N} to deter entry is not beneficial for the society.

5.2 Private firm enters

With entry the game becomes a mixed duopoly competition. Let E denotes the case with entry. The profit functions for firm 1 and 2, are, respectively, $\pi_{1E} = (a - q_1 - q_2 - kc)q_1$ and $\pi_{2E} = (a - q_1 - q_2 - c)q_2$. The per-period social welfare is given as

$$W_E = a(q_1 + q_2) - 1/2(q_1^2 + q_2^2) - q_1 q_2 - kcq_1 - cq_2 - 2F(0)$$

Solving the duopoly game simultaneously for outputs yields $q_{1E} = (a + c - 2kc)$ and $q_{2E} = (kc - c)$, which are referred as the unconstrained level of outputs when both firm coexist in all periods. We get the following lemma:

Corollary 2. *(i) For* $k = 1$*, we have* $q_{1E} = q_{1N}$ *and* $q_{2E} = 0$*; and (ii) for* $k > 1$*, we get* $q_{1E} < q_{1N}$ *and* $q_{2E} > 0$*.*

Corollary 2(i) states that when public are private firms are equally efficient, private firm drops out and the entire market is served by the public firm only (see also footnote 16). On the contrary, Corollary 2(ii) posits that when the public firm is less efficient than the private firm, for each unit of output produced the private firm generates more surplus than that is produced by the public firm. Thus, when the private firm is present in the market, it is optimal for the public firm to restrain its production and let the more efficient private firm to produce more. However, the public firm does not completely stop production simply because even though the private firm produces efficiently, it under produces which is less than the desired social level of output.

5.3 The timing of entry

Up until now nothing has been said about the timing of entry by private firm. Given that public firm enters in period 1, private firm can either enter in period 1 or 2. Let q_{1f} and q_{1s}, denotes the level of output produced by the public firm if private firm enters in period 1 and 2, respectively. The following Lemma holds the conditions for private firm's decision to enter in a particular period.

Lemma 3. *(i) If* $q_{1E} \leq q_{1f} \equiv (a - c) - 2\sqrt{F(0) - \delta F(1)}$*, private firm enters in period 1; (ii) If* $q_{1E} \leq q_{1s} \equiv (a - c) - 2\sqrt{F(1) - \delta F(1)}$*, private firm enters in period 2.*

After laying out conditions of entry for the private firm, we now discuss the conditions for the public firm that determine her production decision. Knowing that public firm is relatively inefficient than private firm, situations may arise that will cause termination of output by public firm. Since private firm can enter in any of the two periods, public firm's decision to produce positive output in either period depends primarily on the size of the inefficiency parameter k. It then follows that there are two possible cut-off levels of inefficiency (k_f and k_s) such that public firm will choose to produce output if and only if $k < k_f$ and $k < k_s$, respectively, for period 1 and 2. The cut-off levels are expressed as:[17]

$$k_f = 1 + \frac{\sqrt{F(0) - \delta F(1)}}{c}$$
$$k_s = 1 + \frac{\sqrt{F(1) - \delta F(1)}}{c}$$

The following corollary is immediate:

Corollary 3. *(i)* $q_{1s} > q_{1f}$*, and (ii)* $k_f > k_s$*.*

Let us intuitively explain the mechanism of this result. The greater the current production by the public firm, the less profits are left for the private firm to exploit and therefore entry will be delayed. Hence, the first period entry which requires greater fixed cost than second period entry will only be carried out by the private firm when the public firm produces a smaller amount. Moreover, a higher level of cost inefficiency on the behalf of the public firm will lead to earlier entry by the private firm.

5.4 Equilibrium entry pattern and welfare under mixed duopoly

Here we analyze the equilibrium entry patterns under mixed duopoly and their implications on the social welfare. There are three possibilities. First, private firm does not enter and the public firm behaves as a monopolist. The reduced-form welfare function is therefore

$$W_N = \frac{1}{2}\frac{(a - kc)^2}{1 - \delta} - F(0) \tag{9}$$

Second, private firm enters in period 1 so that the competition mode is duopoly from hereafter. The reduced-form welfare is given as

$$W_{Ef} = \frac{1}{2}\frac{(a^2 + 3k^2c^2 - 2akc - 4kc^2 + 2c^2)}{1 - \delta} - 2F(0) \tag{10}$$

Finally, private firm decides to enter in period 2. In this case, we have monopoly outcome in period 1 and thereafter duopoly competition from period 2. The reduced-form welfare is

$$W_{Es} = \frac{(a - kc)^2}{2} + \frac{1}{2}\frac{\delta(a^2 + 3k^2c^2 - 2akc - 4kc^2 + 2c^2)}{1 - \delta} - F(0) - \delta F(1) \tag{11}$$

[17]The derivations are provided in the Appendix

If the private firm does not enter in the market, the public firm acts as a monopolist thereby producing output level q_{1N} with the associated welfare W_N (equation 9). But things get complicated when private firm enters in the market. Recall that, $q_{1E} = (a + c - 2kc)$ refers the unconstrained level of output produced by the public firm when both firms coexist in the market. Also, recall that public firm's decision to produce positive output in any period hinges upon the parameter k and demand conditions. We are now in a position to characterize the equilibrium of this game under two possible scenarios.

5.4.1 Case 1: $(a + c - 2kc)|_{k=k_f} > 0$

In this case private firm enters in period 1 and public firm produces positive output. First, observe that at $k = 1$ we have $W_N > W_{Es} > W_{Ef}$, suggesting that public firm is equally efficient and therefore private firm does not enter. In addition, it can be shown that $\frac{\partial W_N}{\partial k} < \frac{\partial W_{Es}}{\partial k} < \frac{\partial W_{Ef}}{\partial k} < 0$ indicating that W_N is declining more steeply than W_{Es} with respect to k (see Figure 3 for an illustration). Thus, when $k \in [1, k_s)$ it follows that $W_N \geq W_{Es}$, suggesting that public firm behaves as a monopolist and therefore no entry takes place. Indeed, for $k = k_s$ we have $W_N = W_{Es}$ while for $k > k_s$ it turns out that $W_N < W_{Es}$.

{*insert Figure 3 here*}

But at $k = k_s$ we have $W_{Es} > W_{Ef}$. However, since W_{Es} is declining more steeply than W_{Ef}, there exists a k such that $W_{Es} = W_{Ef}$. Indeed at $k = k_f$ ensures that $W_{Es} = W_{Ef}$. Moreover, for $k < k_f$ we have $W_{Es} > W_{Ef}$, while for $k > k_f$ we get $W_{Es} < W_{Ef}$. The logic is same as before, for any k that is below k_f, first period entry by private firm requires constrained optimization while beyond that such requirements are not necessary. Indeed, for $k \geq \frac{a+c}{2c}$ it turns out that $q_{1E} = a + c - 2kc \leq 0$. Consequently, it is optimal for the public firm to produce nothing (i.e., $q_{1E} = 0$) and let the market be served only by the private firm.[18]

The results of this subsection are summarized below.

[18] Whether the public firm would shut down or being privatized is a topic left for another paper. Interestingly, in the case of privatization of the public firm, the market would turn to a (private) duopoly as already outlined in Section 3. A related paper in this regard is Anderson *et al.* (1997) that examine the short-run and long-run effects on social welfare due to privatization of the public firm competing against private firms.

Proposition 3. *Assuming* $(a + c - 2kc)|_{k=k_f} > 0$*, public firm should produce according to: (i) for* $k \in [1, k_s)$*, produce* $q_{1N} = (a - kc)$ *and consequently no entry will take place; (ii) For* $k \in [k_s, k_f)$*, produce* $q_1 = (a - kc)$ *in the first period only and continuing from second period produce* $q_1 = (a + c - 2kc)$*, which will induce second period entry by the private firm; (iii) for* $k \in [k_f, \frac{a+c}{2c})$*, produce* $q_1 = (a+c-2kc)$ *in all the periods which will lead to first period entry by the private firm; and (iv) for* $k \in [\frac{a+c}{2c}, \infty)$*, produce* $q_1 = 0$ *in all the periods which again will lead to first period entry by the private firm.*

5.4.2 Case 2: $(a + c - 2kc)|_{k=k_f} < 0$ but $(a + c - 2kc)|_{k=k_s} > 0$

Compared to Case 1, in this case it is not feasible for the public firm to produce any positive output when $k = k_f$. Since $(a + c - 2kc)|_{k=k_f} < 0$, it must be that $\frac{\pi_m}{1-\delta} - F(0) < \frac{\delta\pi_m}{1-\delta} - \delta F(1)$. In other words, even if there is a possibility of earning monopoly profit in the first period, the private firm would delay her entry until the public firm has initiated the initial sunk cost of entry $F(0)$.

As before, for $k \in [1, k_s)$ only the public firm produces and entry does not occur. For $k \in [k_s, \frac{a+c}{2c})$, private firm enters in period 2. As a result, public firm produces $q_{1N} = a-kc$ in period 1 and from period 2 onwards it produces $q_{1E} = a + c - 2kc$. For $k \in [\frac{a+c}{2c}, \frac{a}{c})$, produce $q_1 = a - kc$ since $a - kc \geq 0$. But there would be no output afterwards since $q_{1E} = a + c - 2kc \leq 0$. For $k \in [\frac{a}{c}, \infty)$, produce $q_1 = 0$ in all the periods since $a - kc \leq 0$ and $a + c - 2kc \leq 0$.

Thus, proposition 3 of subsection 5.4.1 can be replicated with minor modifications.

Proposition 4. *If* $(a + c - 2kc)|_{k=k_f} < 0$ *but* $(a + c - 2kc)|_{k=k_s} > 0$*, the public firm should produce according to: (i) for* $k \in [1, k_s)$ *and produce* $q_1 = a - kc$*, resulting no entry; (ii) for* $k \in [k_s, \frac{a+c}{2c})$ *produce* $q_1 = a - kc$ *in the first period and second period on produce* $q_1 = a+c-2kc$ *which will induce second period entry of the private firm; (iii) for* $k \in [\frac{a+c}{2c}, \frac{a}{c})$ *produce* $q_1 = a - kc$ *in the first period and second period on produce* $q_1 = 0$ *which will induce second period entry of the private firm; and (iv) for* $k \in [\frac{a}{c}, \infty)$ *produce* $q_1 = 0$ *in all the periods.*

To sum up, the statements of the above propositions are self-explanatory and we shall not dwell on them. However, it is worth noting how they characterize the behavior of the public firm who cares not only about consumer surplus, but profit of the firms' as well. Therefore, the incumbent public firm is willing to accommodate entry by the rival private firm with a lower cost structure. This makes our contribution different from existing models of mixed duopoly (e.g., Ware (1986)), in which a public firm is unable credibly to induce entry because of its commitment to maximizing social welfare.

6 Extension to the baseline model

Here we present an extension to the baseline model. Traditionally, the role of a public firm has been regarded primarily as a source of allocative efficiency to the society. That is, public firm overproduce which drives down the market price resulting in high consumer surplus. Here we consider another source of efficiency which has not been considered in the literature of mixed oligopoly. In this case, the public firm bears the cost of initial investment $F(0)$ but does not participate in the production activities. The economic rationale for this behavior by public firm is to correct for the market failure that results in underinvestment by firms and to improve the efficiency of public support to the society as a whole. As mentioned previously, market failures associated with underinvestment may result from the situation where one firm can learn from the actions of others. Thus, public firm's role being the initiator of the sunk costs is thus a market response to market failures that prevents firms from producing socially optimal level of output. Clearly, there is gain in efficiency of public support by eliminating entry barriers which in this paper considered as the "strategic efficiency" associated with a public firm. As an example consider a scenario where the public firm is helping to set-up R&D program or the exploration of oil and gas and then leaving the market to private firms. In the OECD countries, public/private partnerships are abound in facilitating the increases in private R&D and market-driven alliances between firms.[19] Next we define the notion of strategic efficiency we study in this section.

Definition 1. *Strategic efficiency (SE) is measured as the difference in welfare between the baseline model and the modified baseline model where the initial investment* $F(0)$ *is paid by the public firm.*

Recall that there were four equilibrium candidates under the baseline model of private duopoly. Below we analyze a modification of the baseline model in which the public firm

[19] See Cervantes (1999) for an overview of the level of public/private partnerships in science and technology in OECD countries. See also Berger (1985) and Daniels and Trebilcock (2002) for an overview of some issues raised by public-private partnerships in U.S. and Canada, respectively.

"kick off" the initial expenditure only and then leaves the market for the private firms for production of outputs. We want to examine the consequences of such policy by the public in regards to the various equilibrium outcomes obtained under the baseline model of entry. In particular, we compare the level of social welfare under baseline and modified baseline model to see the importance of public firm's role as a strategic initiator of the sunk cost. To motivate the analysis, denote by W_{SE} the social welfare under strategic efficiency.

6.1 Simultaneous entry (E_1)

Recall that under equilibrium E_1, both firms enter in period 1. In other words, $\frac{\pi_d}{1-\delta} - F(0) > \delta[\frac{\pi_d}{1-\delta} - F(1)] > 0$. Suppose, the public firm initiates $F(0)$ in period 1. With the initial sunk cost being paid, the first firm to enter gains more profit at the expense of the state since $\frac{\pi_d}{1-\delta} > \frac{\pi_d}{1-\delta} - F(0) > 0$. However, the decision of the second firm is same as before since it remains true that $\frac{\pi_d}{1-\delta} - F(0) > \delta[\frac{\pi_d}{1-\delta} - F(1)] > 0$. Thus, the outcome is similar to the entry situation resulting in a Cournot duopoly. As a result, we get $W_1 = W_{SE}$, implying that there is no gain due to strategic effect under equilibrium E_1 as both firms enter anyway with or without intervention.

6.2 Sequential entry (E_2)

Recall that equilibrium E_2 prevails if $\delta \in [w, 1)$, that is one firm enters in period 1 while the other enters in period 2. Under this case, $[\frac{\pi_d}{1-\delta} - F(0)] < \delta[\frac{\pi_d}{1-\delta} - F(1)]$, $\left[\frac{\pi_d}{1-\delta} - F(1)\right] \geq 0$ and $\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0) \geq 0$. If initial expenditure is incurred by the state instead of the firm, the entry decisions of both firms remain unaltered. The first firm is exempt from the initial expenditure. So, the entry becomes even more preferable: $\pi_m + \delta\frac{\pi_d}{1-\delta} > \pi_m + \delta\frac{\pi_d}{1-\delta} - F(0) \geq 0$. However, it remains valid that $[\frac{\pi_d}{1-\delta} - F(0)] < \delta[\frac{\pi_d}{1-\delta} - F(1)]$ or the late entry is still preferable to the other firm. So, there is no gain from strategic point of view such that $W_2 = W_{SE}$.

6.3 Single entry (E_3)

In the case of equilibrium E_3, there is room for only one firm since $[\frac{\pi_m}{1-\delta} - F(0)] \geq 0$ but $[\frac{\pi_d}{1-\delta} - F(1)] < 0$. Even though the public firm initiates $F(0)$, the profit to the other firm does not change. Thus, there would be only one firm existing in the market since it remains true that $[\frac{\pi_d}{1-\delta} - F(1)] < 0$. Again, there is no gain in efficiency due to strategic effect, therefore $W_3 = W_{SE}$.

6.4 No entry (E_4)

Under equilibrium E_4, neither firm can enter. Recall that there are two possible cases, each of which are being evaluated below.

Case 1: Suppose, $[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)] < 0$ but $[\frac{\pi_d}{1-\delta} - F(1)] > 0$. In this case, if a firm initiates production in period 1, the other firm will inevitably enter in period 2 since the cost of entry is cheaper now. But this will deplete the profit of the pioneering firm and as a result entry does not take place at all in any period.

Suppose, in this context, the public firm invests $F(0)$ and thereafter leaves the market to the private firms at no cost. Now, the first firm would undertake the production happily since profit is positive and the firm need not to pay the initial expenditure or $\pi_m+\delta\frac{\pi_d}{1-\delta} > 0 > \pi_m+\delta\frac{\pi_d}{1-\delta}-F(0)$. However, the other firm would still not enter in the first period since entering simultaneously yield losses as $[\frac{\pi_d}{1-\delta}-F(0)] < [\pi_m+\delta\frac{\pi_d}{1-\delta}-F(0)] < 0$. But since $[\frac{\pi_d}{1-\delta} - F(1)] > 0$, the entry would take place in the second period. Hence, W_{SE} differs from W_4, where

$$W_{SE} = \frac{3(a-c)^2}{8} + \frac{4}{9}\frac{\delta(a-c)^2}{1-\delta} - F(0) - \delta F(1) \tag{12}$$

Case 2: Suppose, $[\frac{\pi_m}{1-\delta} - F(0)] < 0$. In other words, there is not enough room for any firms to enter. Now suppose the initial expenditure is undertaken by the public firm and as a result the entry of the first firm becomes profitable since $\frac{\pi_m}{1-\delta} > 0$. In this case, only one firm will enter if $[\frac{\pi_d}{1-\delta} - F(1)] < 0$. The welfare is therefore

$$W_{SE} = \frac{1}{1-\delta}\frac{3(a-c)^2}{8} - F(0) \tag{13}$$

However, both firms will enter if $[\frac{\pi_d}{1-\delta} - F(1)] > 0$, which yields the same level of welfare as in (12).

We summarize the above results as follows.

Proposition 5. *(i) Under equilibrium E_1, E_2 and E_3, there is no positive gain in efficiency due to strategic effect; and (ii) Under equilibrium E_4 there can be positive gain due to strategic effect as long as $W_{SE} \geq W_4$.*

Proposition 5 states that under equilibrium E_1, E_2 and E_3, the initial investment of the public firm has no effect. By contrast, under equilibrium E_4 the strategic effect can be particularly strong. Under the no-entry case, both firms fail to enter due to mutual suspicion of becoming a loss making pioneer after a delayed successful entry by the other. However, the mere initiation of production by the public firm will induce one firm to enter since it does not have to pay any fixed cost and the other will follow as well in the next period. The strategic effect will be positive if the welfare obtained is positive as well.

7 Conclusion

The presence of a positive externality between investing early and moving late raises the issue of the optimal timing of entry that firms generally face. While moving early generates monopoly profits for the early-mover, the late-mover is also benefits from lower entry costs. This trade-off between entering early or late is the source for multiple equilibria in the model developed here. Thus, for instance, when the benefits of moving late is low firms prefer simultaneous moving in period 1. This situation is overturned when market conditions (i.e., demand) change such that neither firms are ready to enter.

We also analyze the entry game in the context of mixed duopoly comprising of a public and a private firm. We find that equilibrium outcome depends on the level of cost inefficiency associated with the public firm. Finally, we examine the role of the public firm as a strategic initiator where it bears the costly initial expenditure and then leave the market to the private firms who compete in output. Results show that this behavior of public firm can play important role in determining entry timing.

This paper can be extended in many important ways. First, for tractability we confine our analysis to linear demand and quadratic demand functions. It would be interesting to check the robustness of our model under more general demand and cost condition. Second, one can extend the model by allowing several private firms but with the number not large enough so that imperfect competition sustains. Under this framework, it would be worthwhile to examine the entry patterns and find out how the presence of a public firm influences the entry timing. Third, the learning in this model is associated with initial investment only, but it may be linked to marginal cost as well. It will be interesting to investigate how that will change the prediction of the model. Fourth, another line of extension could be to incorporate non-pecuniary externalities of both static and dynamic types to see how the results alter with the arrival of a public firm. Finally and importantly, our analysis can also be extended along the line of patent licensing literature.[20] Whereas the present analysis assumes that there are no property rights over the innovations of the leader. An interesting area for future research would be how a patent regime would influence the entry patterns. As is well known that patents generally have terms; clearly in the case with long and enforceable patent, the entry decision will differ significantly between private and mixed oligopoly.

8 Appendix

The appendix contains all derivations and proofs not included in the text.

8.1 Proof of Lemma 1

Suppose, both firms enter in period $t+1 > 1$. But one firm can easily do better by entering earlier since $\delta^t[\pi_m + \delta\frac{\pi_d}{1-\delta} - F(0)] > \delta^{t+1}[\frac{\pi_d}{1-\delta} - F(1)]$. Thus, if both decide to enter in period t, they must do it in period 1. This will be true if $\delta < \frac{\pi_m + \frac{\pi_d}{1-\delta} - F(0)}{\frac{\pi_d}{1-\delta} - F(1)} \equiv w$.

8.2 Proof of Lemma 2

Immediate.

8.3 Proof of Lemma 3

(i) A private firm will enter in the first period if the following conditions hold: (i) Profit of the firm entering in the first period must be positive, i.e. $\frac{1}{4}\frac{(a-q_{1E}-c)^2}{1-\delta} \geq F(0)$. In other words, $q_{1E} \leq (a-c) - 2\sqrt{(1-\delta)F(0)}$; (ii) It is profitable to go to the first period rather than wait for the second, $\frac{(a-q_{1E}-c)^2}{4} \geq [F(0) - \delta F(1)]$ or, $q_{1E} \leq (a-c) - 2\sqrt{F(0) - \delta F(1)}$. In other words, to ensure entry in the first period, we need

$$q_{1E} \;\leq\; \text{Min}[(\text{a} - \text{c}) - 2\sqrt{(1-\delta)\text{F}(0)},\quad (\text{a} - \text{c}) - 2\sqrt{\text{F}(0) - \delta\text{F}(1)}]$$

[20]See Kamien (1992) for an overview of the literature on patent licensing.

implying $q_{1E} \leq (a-c) - 2\sqrt{F(0) - \delta F(1)}$. Since, $[F(0) - \delta F(1)] > [F(0) - \delta F(0)]$, we define $q_{1f} \equiv (a-c) - 2\sqrt{F(0) - \delta F(1)}$, thus providing Lemma 4(i).

(ii) If there is entry in the second period, the welfare is, $W_{Es} = W_N + \frac{\delta}{1-\delta} W_{Es}$. Since, there is no entry in the first period, the welfare maximizing output is q_{1N}. The minimum output required for profitable entry in the second period requires, $\frac{1}{4}\frac{(a-q_{1E}-c)^2}{1-\delta} \geq F(1)$. In other words, $q_{1E} \leq (a-c) - 2\sqrt{(1-\delta)F(1)}$. We define $q_{1s} \equiv (a-c) - 2\sqrt{(1-\delta)F(1)}$.

8.4 Proof of Corollary 1 to 3

Immediate.

8.5 Derivation of k_f and k_s.

The derivations of k_f and k_s follow directly from substituting for the outputs. For $q_{1E} \leq q_{1f}$, it follows that $(a + c - 2kc) < (a-c) - 2\sqrt{F(0) - \delta)F(1)}$, solving for k yields $k_f \equiv 1 + \frac{\sqrt{F(0)-\delta F(1)}}{c}$. The other cut-off point k_s is derived in a similar way.

8.6 Proof of Proposition 1 to 5

Immediate.

Figure 1: Equilibrium conditions under baseline model.

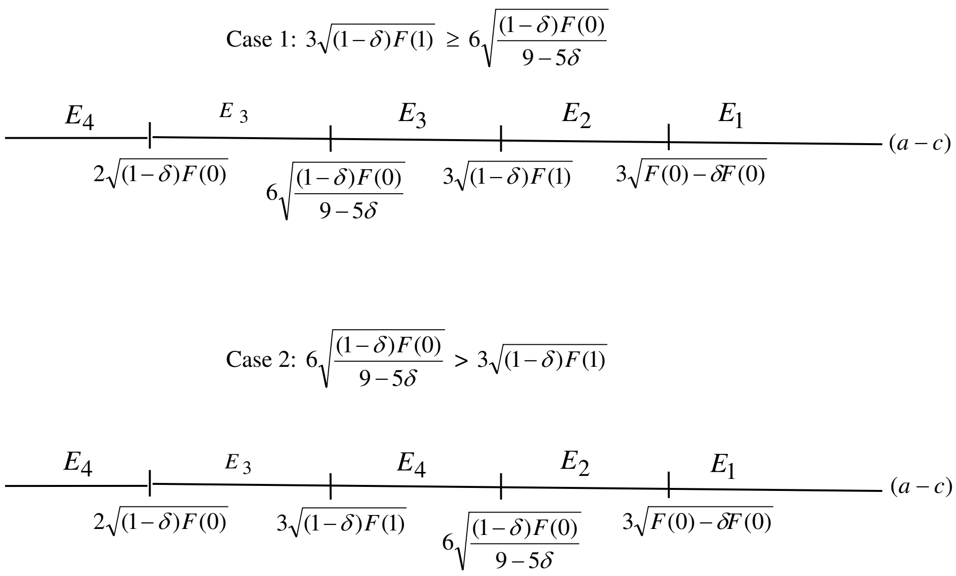

Figure 2: Entry under mixed duopoly.

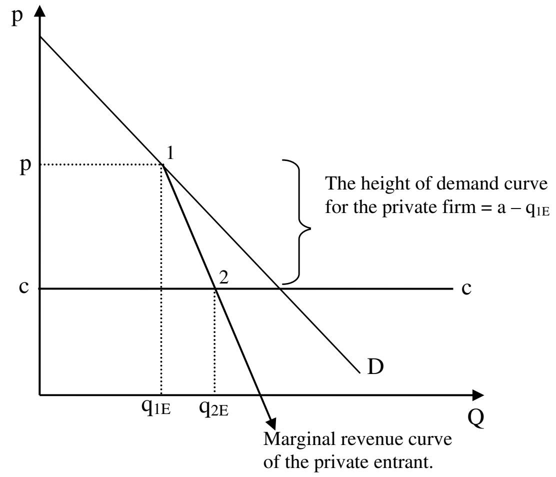

Figure 3: Social welfare under mixed duopoly with different cost inefficiency parameters

This figure plots the welfare functions obtained under mixed duopoly game for the following range of the cost inefficiency parameter (k=1 to 6). The following parameters were used to generate Figure 3: $a = 10$, $c = 1$, $F(0) = 50$, $F(1) = 40$, and $\delta = 0.9$. Notations: W_N represents the social welfare in the case where private firm does not enter and the market structure is a monopoly with public firm being the sole producer. W_{Ef} (W_{Es}) denotes the case where the private firm enters in the first (second) period. As can be seen, $W_N = W_{Es}$ when $k = k_s$. As the the inefficiency parameter gets higher, W_{Es} and W_{Ef} begin to converge and become equal at $k = k_f$.

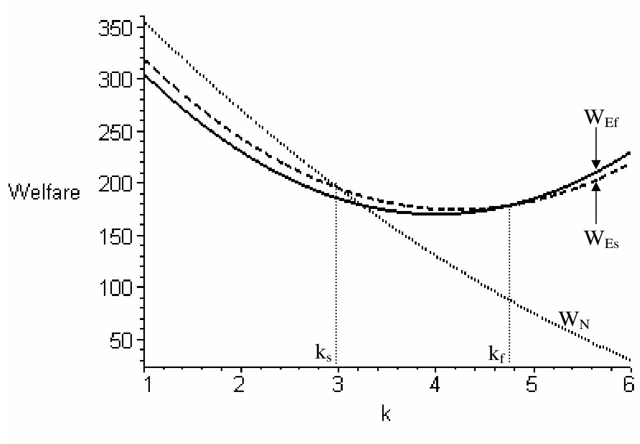

Figure 3: Social welfare under mixed duopoly with different cost inefficiency parameters.

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