A NOTE ON INNOVATION BY LEADERS WITHOUT WINNER-TAKE-ALL

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Abstract: In innovative races with winner takes all, leading firms invest less than each follower, given exogenous entry (Reinganum, 1985). But with endogenous entry this result is reversed (Etro, 2004). It is argued here that sharing of rewards between the players may alter these predictions.

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1. Introduction

A central feature of the neo-Schumpeterian analysis is the focus on the innovative role of first movers and followers. In so called race settings with technological uncertainty, winner-take-all and a given number of firms, the leader invests less than each follower by which the follower is most likely to win the race (Reinganum, 1985). This finding is consistent with case studies where market-dominating firms tend to be slow in developing important new products, but to roar back like tigers when smaller rivals – often entrants with no historic market share at all – challenge their dominance (Scherer, 1992). The leader entrant engages in less effort than the follower large incumbent. A similar pattern may be typical of cases where imitators are able to surpass pioneers because of better knowledge building (Schnaars, 1994). It is also consistent with the existence of second mover advantages when (expected) profits of the followers exceed (expected) profits of the leader (Reinganum, 1985).

But clearly this can not be the general story. The set of cases with first-mover advantages can not be empty and followers or imitators are often engaged in less R&D effort than the leaders. Many factors can play a role in the reversal of these comparisons. Etro (2004) has shown that with endogenous entry a first mover engages in more efforts than each of the followers, given winner-take-all. In this case a leading firm anticipates that the equilibrium number of entrants will be affected by its own efforts.

In this note it will be argued that first – or second mover advantages and the comparison of leader-follower efforts is also driven by the market sharing possibilities. Winner-take-all settings are important in markets with effective patent protections or with vertical differentiation that leads buyers to the best product, ignoring substitutes that may be very good, but not quite as good as the winner version. But in other cases players will have to share payoffs because of spillovers.

The analysis of asymmetric spillovers and role playing has only recently been touched upon in rather specific situations. Reinganum (1985) and Etro (2004) look at stochastic development efforts of one leader and many followers in a race setting with no attention to output rivalry and spillovers. Results on innovative races without winner-take-all have been analyzed by Stewart (1983) in a setting with simultaneous moves of all rivals. Here two stochastic leader-follower settings with asymmetric market sharing is looked at with both exogenous and endogenous entry.

In one case a winning follower can keep the whole market, but a winning leader may have to share with all other followers. In a second case a winning leader keeps the whole market, but a winning follower may have to share with other followers. This sharing may alter the predictions described above. With exogenous entry, the leader sometimes spends more resources on R&D than the followers and the second movers sometimes invest more than the leader with endogenous entry. And with exogenous entry, the leader may be better off than (one of) the follower(s) so that a first-mover advantage appears.

The remainder of this article proceeds as follows. In section 2, the model and the different cases are presented in detail. In section 3, the efforts of the leader and followers are compared. Section 4 deals with the leader-follower comparison of expected profits. A conclusion is given in section 5. The appendix provides the reader a detailed overview of the solution of the model.

2. The model

Firm L is a Stackelberg leader in the innovative race and firms 1,2,…,n are entrants in the industry and followers. This leader could be seen as a current patent holder who obtains a flow of profits π due to the current patent, with $\pi \geq 0$. The development process is stochastic with the probability of success of any firm j by time t being equal to $1 - e^{-h(x_j) \times t}$, where $h(x_j)$ is the development intensity and x_j is the development intensity selected by firm j, at a cost of x_i euros per unit of time, with j=L,1,2,..,n. Moreover, $h'(.)>0$ and $h''(.)<0$[1]. F is a fixed cost for each player, with $\sigma \times P - F > 0$. In general, the expected value of discounted profits for the leader with interest rate r is:

$$V^L = \frac{h\left(z^L\right) \times P_1^L + \sum_{j=1}^{n} h\left(z^j\right) \times P_2^L + \pi - z^L}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \qquad (1)$$

and for each of the entrants i=1,2, …,n:

[1] Throughout this paper, numerical examples, performed by Maple, are often used to sketch tendencies. Two different hazard functions are used in these examples, namely $h_1(x) = \sqrt{x}$ and $h_2(x) = 1 - e^{(-x)}$. Both functions satisfy the conditions on the first and second derivative.

$$V^i = \frac{h\left(z^i\right) \times P_1^i + h\left(z^L\right) \times P_2^i + \sum_{\substack{j=1 \\ j \neq i}}^{n} h\left(z^j\right) \times P_3^i - z^i}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \qquad (2)$$

with P_1^L, P_2^L, P_1^i, P_2^i and P_3^i as indicated in table 1. The discussion hereafter focuses on parameter values with $V^L>0$ and $V^i \geq 0$.

Let σ be a market sharing parameter ($\sigma \leq 1$). Two cases with asymmetric sharing define the minimum value of σ and the notation of payoffs, see Table 1.

Technological and market spillovers need not be symmetric between innovative players (Amir and Wooders, 1999). Some firms may be better at protecting their knowledge and markets through raising barriers to imitation and effectively exploring legal instruments such as patents. If they win a race, they will dominate the market. Others that win may not have these abilities and may face imitation by others.

In case A, it is a follower who keeps the market if it wins. If the leader wins, it *may* have to share with all followers. In case B, it is the leader who keeps the whole market if it wins. A follower who wins the race *may* have to share with the other followers.

One leader and n entrants (followers)		A Leaders may share with all entrants	B Sharing may occur among all entrants
		Winner takes all: $\sigma = 1$ Equal sharing: $\sigma = \frac{1}{1+n}$	Winner takes all: $\sigma = 1$ Equal sharing: $\sigma = \frac{1}{n}$
Payoff leader	P_1^L (leaders wins)	$\sigma \times P$	P
	P_2^L (entrant wins)	0	0
Payoff entrant i	P_1^i (entrant wins)	P	$\sigma \times P$
	P_2^i (leader wins)	$\left(\frac{1-\sigma}{n}\right) \times P$	0
	P_3^i (other entrant wins)	0	$\left(\frac{1-\sigma}{n-1}\right) \times P$

Table 1. Payoffs in two races with asymmetric market sharing.

Combining (1), (2) and table 1 yields the specific expected profit functions for case A and case B. These are summarized in Table 2.

Expected Profit Functions	Leader	Follower i i=1,…,n
Case A	$V^L = \dfrac{h\left(z^L\right) \times \sigma \times P + \pi - z^L}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F$	$V^i = \dfrac{h\left(z^i\right) \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z^i}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F$
Case B	$V^L = \dfrac{h\left(z^L\right) \times P + \pi - z^L}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F$	$V^i = \dfrac{h\left(z^i\right) \times \sigma \times P + \sum_{j=1, j\neq i}^{n} h\left(z^j\right) \times \left(\frac{1-\sigma}{n-1}\right) \times P - z^i}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F$

Table 2. Expected profit functions of the leader and the followers in case A and case B.

In the next section, the different games are solved and leader and follower efforts are compared.

3. Comparing leader and entrant efforts

In this section, it is analyzed if the findings of Reinganum (1985) and Etro (2004) still apply when (asymmetric) market sharing is taken into account.

The two different racing games are both solved by backward induction. So, first, profits of the followers are maximized. A symmetric equilibrium is looked at with $z_i=z$, for all i. To continue, the optimal R&D investment of the leader is calculated. More details on solving the games can be found in appendix. Concavity and existence conditions are assumed to be satisfied.

In order to compare leader and follower efforts, a function g.(x) is defined with $g(z^L)=0$ and $g'(.)<0$, in view of the concavity of V^L. The sign of this function determines the comparison of R&D investments of the first and second movers. These functions, for case A and case B, are the following:

Case A.

$$g_A(x) = \begin{array}{l}\left[h'(x) \times \sigma \times P - 1\right] \times \left[r + n \times h(x) + h\left(z^L\right)\right] \\ -\psi^A \times \left[h(x) \times \sigma \times P + \pi - x\right]\end{array} \quad (3)$$

$$\text{with } \psi^A = h'(x) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}. \quad (4)$$

Case B.

$$g_B(x) = \left[h'(x) \times P - 1\right] \times \left[r + n \times h(x) + h\left(z^L\right)\right] - \psi^B \times \left[h(x) \times P + \pi - x\right] \quad (5)$$

$$\text{with } \psi^B = h'(x) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}. \quad (6)$$

Moreover, As in Reinganum (1985), the following stability condition is assumed:

$$\psi^{\bullet} = h'(z) \times \left(1 + \frac{\partial\,(n \times z)}{\partial z^L}\right) > 0. \quad (7)$$

Now, it is possible to compare leader and follower efforts by deriving the sign of $g_{.}(z)$. With $g_{.}(z)>0$, the leader invests more than each follower. With $g_{.}(z)<0$, the reverse applies.

3.1. Exogenous entry

First, the efforts of the leader and the followers are compared when the number of followers is exogenous. Proposition 1 summarizes the main finding. Obviously, π is set equal to zero in order to make the comparison with Reinganum (1985) meaningful.

> Proposition 1: With exogenous entry and no market sharing (σ=1), the leader invests less than each follower (Reinganum, 1985). But with market sharing (σ<1), this tendency may be reversed in case B.

In case B, a winning leader keeps the whole market. A winning follower may have to share with all other followers and this tends to reduce their individual efforts. A numerical example illustrates the tendencies of case A and confirms the reversal in case B. Let P=150, F=1, π=0, r=0,10, n=4 and let $h(x) = \sqrt{x}$.

In case A:

$\sigma = 1$	$\rightarrow$	$z^L = 4325$	$<$	z=4442
$\sigma = 0,75$	$\rightarrow$	$z^L = 2589$	$<$	z=4275
$\sigma = 0,5$	$\rightarrow$	$z^L = 1230$	$<$	z=4156
$\sigma = 0,25$	$\rightarrow$	$z^L = 329$	$<$	z=4101

In case B:

$\sigma = 1$	$\rightarrow$	$z^L = 4325$	$<$	z=4442
$\sigma = 0,75$	$\rightarrow$	$z^L = 3872$	$>$	z=2168
$\sigma = 0,5$	$\rightarrow$	$z^L = 3062$	$>$	z=747
$\sigma = 0,25$	$\rightarrow$	$z^L = 1482$	$>$	z=98

3.2. Endogenous entry

In a long run symmetric equilibrium, the expected profits of the followers are assumed to equal zero, $V^i(z,z^L,n^*)=0$, with n* the endogenous number of entrants. The latter assumption is called the zero profit condition (ZPC). Combining ZPC with the first order condition for an entrant allows verifying that:

Case A: $h'(z) \times (P - F) = 1$ (8)

Case B: $h'(z) \times (\sigma \times P - F) = 1$ (9)

From (3) and (4), it is clear that, in both cases, R&D investments of the followers (z) are independent of the efforts of the leader z^L, which is also the case in the winner-take-all setting of Etro (2004). The leader will, however, anticipate that n* may be influenced by its own effort.

In addition, with endogenous entry, it is known that $\frac{\partial z}{\partial z^L} = 0$ (in both cases, see (8) and (9)) and

$\psi^{\bullet} = h'(z) + h'(z) \times \frac{\partial n^*}{\partial z} > 0$, so

$$\psi^A = h'(x) + \frac{h(z) \times h'(x) \times \left[(1-\sigma) \times V - n^* \times F\right]}{h(x) \times \left(\frac{1-\sigma}{n}\right) \times V + n^* \times F \times h(z)}, \tag{10}$$

$$\psi^B = h'(x) - h'(x) = 0. \tag{11}$$

Now, the sign of $g_{.}(z)$ can be determined.

> Proposition 2: With endogenous entry and no market sharing (σ=1), the leader invests more than each follower (Etro, 2004). With market sharing (σ<1), this tendency remains valid in case B but may be reversed in case A.

In case A, only a winning follower keeps the whole market. A winning leader may have to share with all followers. This tends to discourage its efforts, even if higher investments could reduce the number of imitators, especially if it especially if it expects to share a lot, in other words, if the spillover from leader to followers are substantive. A numerical example[2] confirms this reversed prediction in case A and illustrates the tendency of case B. Let P=150, F=10, π=100, r=0,10 and $h(x) = 1 - e^{-x}$. The ex ante stream of profits, π, is set

[2] Integer constraint on n is ignored here.

positive here in order to guarantee positive expected profits of the leader.

In Case A:

$\sigma = 1$	$\rightarrow$	$z^L = 5,011 > z = 4,942$	and	$n^* = 13,401$
$\sigma = 0,75$	$\rightarrow$	$z^L = 4,686 < z = 4,942$	and	$n^* = 13,677$
$\sigma = 0,5$	$\rightarrow$	$z^L = 4,230 < z = 4,942$	and	$n^* = 13,943$

In Case B:

$\sigma = 1$	$\rightarrow$	$z^L = 5,011 > z = 4,942$	and	$n^* = 13,401$
$\sigma = 0,75$	$\rightarrow$	$z^L = 5,011 < z = 4,630$	and	$n^* = 13,428$
$\sigma = 0,5$	$\rightarrow$	$z^L = 5,011 < z = 4,174$	and	$n^* = 13,466$

These tendencies of case A were also found in a strategic investment game with leaders and followers, characterized by no uncertainty and exogenous entry (Vandekerckhove and De Bondt, forthcoming). Large spillovers from a leader to imitators may result in them investing less than each follower, even though a large effort could improve their subsequent Stackelberg profits.

4. Comparing leader and follower profits

In this section, the profits of the leader and an entrant are compared. This exercise needs only to be done with exogenous entry. Since the expected profits of the leader are assumed to be strict positive and with endogenous entry, the zero profit condition states that the expected profits of the followers equal zero. Proposition 3 summarizes the main findings.

Proposition 3

With exogenous and winner-take-all (σ=1) and π=0, expected profits of the leader are always smaller than expected profits of each follower (Reinganum, 1985). But with market sharing ($\sigma<1$) and $\pi=0$, this prediction remains valid in case A, but *may* be reversed in case B (with σ small enough).

Followers always benefit from second mover advantages in case A. A numerical example illustrates this tendency. Let P=150, F=1, $\pi=0$, r=0.10, n=4, and let $h(x)=\sqrt{x}$.

$\sigma=1 \quad \rightarrow \quad V^L=15.664 < V=15.710$

$\sigma=0,75 \quad \rightarrow \quad V^L=9.032 < V=18.229$

$\sigma=0,5 \quad \rightarrow \quad V^L=3.779 < V=20.060$

$\sigma=0,25 \quad \rightarrow \quad V^L=0.280 < V=20.921$

In case B, however, the leader may benefit, in terms of expected profits, from moving first in the R&D stage. The winning leader takes it all and the winning follower needs to share with other followers, it could be that leaders' expected profits are higher than the followers' expected profits. After all, the market sharing of followers tends to reduce their efforts and hence their profitability. Market sharing can, in other words, alter the prediction of Reinganum (1985). Again, a numerical example sets this tendency. Let P=150, F=1, $\pi=0$, r=0.10, n=4 and let $h(x)=\sqrt{x}$.

$\sigma=1 \quad \rightarrow \quad V^L=15.664 < V=15.710$

$\sigma=0,75 \quad \rightarrow \quad V^L=20.973 > V=18.376$

$\sigma=0,5 \quad \rightarrow \quad V^L=30.798 > V=19.349$

$\sigma=0,25 \quad \rightarrow \quad V^L=53.932 > V=16.726$

5. Conclusion

Since competition and industry policies can focus on leaders and/or followers it is important to have a good understanding of their incentives and roles in innovative activities. The incorporation of asymmetric spillovers can help the search for richer hypotheses to be tested in empirical work.

In this paper, patent races are analyzed in which the winner of the does not necessarily takes the full value of the discovery. Both exogenous and endogenous entry are looked at. When the number of firms is taken as given (short run perspective) and a winning follower may have to share with the other followers while a winning leader keeps the whole market, it could be that the leader invests more than each of the followers. Moreover, in this case, it is also possible that the expected profits of the leader are higher than the expected profits of the followers.

When the number of followers is endogenous (long run perspective), a follower may invest more than the leader. This tends to happen in situation in which a winning leader may have to share with the followers while a winning follower takes it all.

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Appendix

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The expected value of discounted profits for the leader:

$$V^L = \frac{h\left(z^L\right) \times P_1^L + \sum_{j=1}^{n} h\left(z^j\right) \times P_2^L + \pi - z^L}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \,. \tag{A1}$$

The expected value of discounted profits for each follower:

$$V^i = \frac{h\left(z^i\right) \times P_1^i + h\left(z^L\right) \times P_2^i + \sum_{\substack{j=1 \\ j\neq i}}^{n} h\left(z^j\right) \times P_3^i - z^i}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \,. \tag{A2}$$

Case A

Combining Table 1 with (A1) and (A2) results in the following expected profits functions for the leader and for each entrant in case A.

$$V^L = \frac{h\left(z^L\right) \times \sigma \times P + \pi - z^L}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \tag{A3}$$

$$V^i = \frac{h\left(z^i\right) \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z^i}{r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)} - F \tag{A4}$$

For each of the n entrants, the first order condition needs to be satisfied, $\frac{\partial V^i}{\partial z^i} = 0$.

$$\frac{\partial V^i}{\partial z^i} = \frac{\left[h'\left(z^i\right) \times P - 1\right] \times \left[r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)\right] - \left[h'\left(z^i\right)\right] \times \left[h\left(z^i\right) \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z^i\right]}{\left[r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)\right]^2} = 0 \tag{A5}$$

In a symmetric equilibrium, the following applies

$$\phi^i = \left[h'(z) \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \left[h'(z)\right] \times \left[h(z) \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z\right] = 0 \tag{A6}$$

The sign of $\frac{dz}{dz^L}$ is equal to the sign of $\left[\left(r + n \times h(z)\right) \times \sigma \times P - n \times z - r \times P\right]$. So for $\sigma < \sigma°$, the efforts are strategic substitutes but for $\sigma° < \sigma \leq 1$, the efforts are strategic complements, with $\sigma° = \frac{r \times P + n \times z}{r \times P + n \times h(z) \times P}$.

The leader maximizes its profits by choosing z^L. Thus,

$$\frac{\partial V^L}{\partial z^L} = \frac{\left[h'\left(z^L\right) \times \sigma \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \left[h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}\right] \times \left[h\left(z^L\right) \times \sigma \times P + \pi - z^L\right]}{\left[r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)\right]^2} = 0 \tag{A7}$$

or

$$\phi^L = \left[h'\left(z^L\right) \times \sigma \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \left[h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}\right] \times \left[h\left(z^L\right) \times \sigma \times P + \pi - z^L\right] = 0 \tag{A8}$$

Now define the following function:

$$g_A(x) = \left[h'(x) \times \sigma \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \psi^A \times \left[h(x) \times \sigma \times P + \pi - x\right] \tag{A9}$$

with $g_A\left(z^L\right) = 0$ and $\psi^A = h'(x) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}$

Then,

$$g'(x) = \frac{\partial^2 V^L}{\partial x^{L^2}} - h'(z) \times \left(h'\left(z^L\right) \times \sigma \times P - 1\right) < 0 \tag{A10}$$

Combining (A9) and (A6) and evaluating in x=z,

$$g(z) = -h'(z) \times (1-\sigma) \times P \times \left(r + n \times h(z) + h\left(z^L\right)\right) - \psi^A \times \left[h(z) \times \sigma \times P + \pi - z\right] + h'(z) \times \left[h(z) \times \sigma \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z\right] \tag{A11}$$

The sign of (A11) drives the comparison of z and z^L.

- Exogenous entry

  - With $\sigma = 1$, $g_A(z)\big|_{\sigma=1} = -\frac{\partial\left(n \times h(z)\right)}{\partial z^L} \times \left[h(z) \times P + \pi - z\right] - h'(z) \times \pi < 0$ (A12)

    Since z and z^L are strategic complements for $\sigma = 1$, $\frac{\partial z}{\partial z^L} > 0$, from A6.

  - The sign of $g_A(z)$ for $\sigma < 1$ is unclear in general. Numerical analysis suggests that in a wide range of cases, the sign remains negative.

- Endogenous Entry

With endogenous entry, the zero profit condition states that $V^i = 0$ for i=1,2,…,n. From (A4):

$$ZPC = h(z) \times P + h(z^L) \times \left(\frac{1-\sigma}{n}\right) \times P - z - F \times \left[r + n \times h(z) + h(z^L)\right] = 0 \qquad \text{(A13)}$$

Combining (A6) and (A13) yields

$$h'(z) \times (P - F) = 1 \qquad \text{(A14)}$$

From (A14), z can be derived and it is clear that z is not dependent on z^L.

From the Zero Profit Condition of the followers, $\psi^A = h'(z^L) + \frac{\partial(n \times h(z))}{\partial z^L}$ can be calculated, as

$$\psi^A = h'(z^L) + \frac{\partial(n \times h(z))}{\partial z^L} = h'(z^L) + \frac{\partial n}{\partial z^L} \times h(z) + n \times h'(z) \times \frac{dz}{dz^L} = h'(z^L) + \left(-\frac{\partial ZPC/\partial z^L}{\partial ZPC/\partial n}\right) \times h(z) + 0 = h'(z^L) + \frac{h'(z^L) \times \left[\left(\frac{1-\sigma}{n}\right) \times P - F\right]}{\frac{h(z^L)}{h(z)} \times \left(\frac{1-\sigma}{n^2}\right) \times P + F}. \qquad \text{(A15)}$$

Finally, A(15) should be introduced in A(11). For $\sigma = 1$, $g(z) > 0$, thus $z^L > z$. For $\sigma < 1$, the sign of g(z) can be positive or negative.

Case B

Combining Table 1 with (A1) and (A2) results in the following expected profits functions for the leader and for each entrant in case A.

$$V^L = \frac{h(z^L) \times P + \pi - z^L}{r + \sum_{j=1}^{n} h(z^j) + h(z^L)} - F \tag{A16}$$

$$V^i = \frac{h(z^i) \times \sigma \times P + \sum_{\substack{j=1 \\ j \neq i}}^{n} h(z^j) \times \left(\frac{1-\sigma}{n-1}\right) \times P - z^i}{r + \sum_{j=1}^{n} h(z^j) + h(z^L)} - F \tag{A17}$$

For each of the n entrants, the first order condition needs to be satisfied, $\frac{\partial V^i}{\partial z^i} = 0$.

$$\frac{\partial V^i}{\partial z^i} = \frac{\left[h'(z^i) \times \sigma \times P - 1\right] \times \left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right] - \left[h'(z^i)\right] \times \left[h(z^i) \times \sigma \times P + \sum_{\substack{j=1 \\ j \neq i}}^{n} h(z^j) \times \left(\frac{1-\sigma}{n-1}\right) \times P - z^i\right]}{\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right]^2} = 0 \tag{A18}$$

In a symmetric equilibrium, the following applies

$$\phi^i = \left[h'(z) \times \sigma \times P - 1\right] \times \left[r + n \times h(z) + h(z^L)\right] - \left[h'(z)\right] \times \left[h(z) \times P - z\right] = 0 \tag{A19}$$

The leader maximizes its profits by choosing z^L. Thus,

$$\frac{\partial V^L}{\partial z^L} = \frac{\left[h'\left(z^L\right) \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \left[h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}\right] \times \left[h\left(z^L\right) \times P + \pi - z^L\right]}{\left[r + \sum_{j=1}^{n} h\left(z^j\right) + h\left(z^L\right)\right]^2} = 0 \tag{A20}$$

or

$$\phi^L = \left[h'\left(z^L\right) \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \left[h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}\right] \times \left[h\left(z^L\right) \times P + \pi - z^L\right] = 0 \tag{A21}$$

Now define the following function:

$$g_B(x) = \left[h'(x) \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] - \psi^B \times \left[h(x) \times P + \pi - x\right] \tag{A22}$$

with $g_B\left(z^L\right) = 0$ and $\psi^B = h'(x) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}$

Then,

$$g_B'(x) = \frac{\partial^2 V^L}{\partial x^{L^2}} - h'(z) \times \left(h'\left(z^L\right) \times P - 1\right) < 0 \tag{A23}$$

The sign of (A22) drives the comparison of z and z^L.

- Exogenous entry

  - With $\sigma = 1$, $g_B(z)\big|_{\sigma=1} = -\psi^B \times \left[h(z) \times P + \pi - z\right] < 0$, by which $z^L < z$.

  - With $\sigma < 1$, it is possible that $g_B(z) > 0$, by which $z^L > z^F$. The example in the text is a proof of this.

- Endogenous entry

  With endogenous entry, the zero profit condition states that $V^i = 0$ for i=1,2,…,n. From (A17)

$$ZPC = h(z) \times P - z - F \times \left[r + n \times h(z) + h\left(z^L\right)\right] = 0 \qquad \text{(A24)}$$

  Combining (A19) and (A24) yields

$$h'(z) \times (\sigma \times P - F) = 1 \qquad \text{(A25)}$$

  From (A26), z can be derived and it is clear that z is not dependent on z^L.

  From the Zero Profit Condition of the followers, $\psi^B = h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L}$ can be calculated, as

$$\psi^B = h'\left(z^L\right) + \frac{\partial\left(n \times h(z)\right)}{\partial z^L} = h'\left(z^L\right) + \frac{\partial n}{\partial z^L} \times h(z) + n \times h'(z) \times \frac{dz}{dz^L} = h'\left(z^L\right) + \left(-\frac{\partial ZPC/\partial z^L}{\partial ZPC/\partial n}\right) \times h(z) + 0 = h'\left(z^L\right) + h(z) \times \left(-\frac{-h'\left(z^L\right) \times F}{-h(z) \times F}\right) = 0. \quad \text{(A26)}$$

Consequently,

$g_B(z) = \left[h'(z) \times P - 1\right] \times \left[r + n \times h(z) + h\left(z^L\right)\right] > 0$, by which $z^L > z$ for $\frac{1}{n} \leq \sigma \leq 1$. (A27)

<u>Proof of proposition 3</u>

From the first order condition of the leader (A8) and the followers, it is known that $h\left(z^L\right) \times \sigma \times P - z^L$ is an increasing function in z^L and $h(z) \times P - z$ is an increasing function z.

$$z^L < z \text{ (From proposition 1)}$$
$$\Downarrow$$
$$h\left(z^L\right) \times P - z^L < h(z) \times P - z$$
$$\Downarrow$$
$$h\left(z^L\right) \times \sigma \times P - z^L < h(z) \times P - z$$
$$\Downarrow$$
$$h\left(z^L\right) \times \sigma \times P - z^L < h(z) \times P + h\left(z^L\right) \times \left(\frac{1-\sigma}{n}\right) \times P - z$$