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INNOVATION BY LEADERS II: SCHUMPETERIAN GROWTH WITH ENDOGENOUS PERSISTENCE OF LEADERSHIP*

ABSTRACT

I develop a Schumpeterian model of endogenous growth with realistic features of the market for innovations as decreasing marginal productivity at the firm level and the possibility of wasteful duplications of resources between firms due to congestion at the industry level. Moreover, I consider the possibility that incumbent patentholders have a competitive advantage in the patent races for the next generation technologies and hence endogenously invest in R&D: in this case the value of being a leader is higher and growth driven by market leaders is higher. Technically the paper provides a complete analytical solution of the general equilibrium model with endogenous persistence of leadership through dynamic programming techniques and undetermined coefficient methods. Moreover, this framework can be used for other macroeconomic investigations: I show that other sources of growth may reduce investment inducing a paradoxical negative correlation between growth and R&D spending, and that price stickiness induces an inverted U relation between inflation and long run growth.

Key words: Growth, Innovation.
JEL Classification: O3, O4, F4.

I - INTRODUCTION

According to Schumpeter (1942), market power provides the incentives to invest and innovate and so it stimulates growth. The recent revival of this important observation, started with the works by Romer (1990) and Aghion - Howitt (1992), has formalized this idea of creative destruction. However, the market for innovations is usually described in a very simple and empirically arguable way: the production function of new ideas is characterized by constant marginal productivity and a simple no-arbitrage condition pins down the equilibrium investment in R&D and, consequently, the rate of economic growth. This minimalistic approach does not allow to open the «black box» of the engine of growth, its microeconomic organization, and to study how policy affects it.

As noticed by Kortum (1993), Cohen - Klepper (1996) and others, investments in

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R&D are characterized by relevant fixed costs, decreasing marginal productivity[1], and substantial duplications of resources between firms. In a companion paper, Etro (2004), I have studied a partial equilibrium model of patent races with alternative kinds of competition and I have introduced it in a sketched Schumpeterian growth model.[2] In the current article I expand, correct and complete that exercise to develop a fully fledged general equilibrium model of growth with realistic patent races in the market for innovations and to study its welfare implications. The most important outcome is that, even if the equilibrium can be characterized by a growth rate below or above the optimal one, the organization of the market for innovation is always biased toward too small firms[3]. Notice that this general conclusion may shed new light on the problems of countries that do not grow much, are characterized by many small firms investing too little in R&D and lack large and innovative corporations, notably Italy.

Another important stylized fact about R&D investment is that a large part of it is done by incumbent monopolists with patents on the leading edge technologies (see Blundell - Griffith - Van Reenen, 1999; Aghion - Griffith, 2005)[4]. It has been claimed that the engine of global growth is actually in those large corporations leading research in high-tech sectors which mostly contribute to productivity growth. This important stylized fact is neglected by the entire literature on Schumpeterian growth, where incumbent monopolists do not invest in R&D according to the traditional theory of innovation (Arrow, 1962). In Etro (2002, 2004) I have provided a new rationale for investment in R&D by monopolists: the crucial conditions are a leadership of the monopolists and free entry in the patent race for the next technology. In a dynamic context, this leads to persistence of monopolies and hence it increases the value of innovations. In particular, when marginal productivity is close to constant, monopolists deter entry in the patent race remaining the only innovators,

[1] One of the stilyzed facts pointed out by Cohen - Klepper (1996) is that the number of patents and innovations per dollar of R&D decreases with the level of R&D. See Griliches (1994) for an empirical discussion. From a theoretical point of view, notice that, while in most of the productive sectors there are strong reasons to believe that doubling the amount of input total production will double, there no any reasons to believe that doubling the amount of input in the R&D activity will deliver a double expected amount of innovations or will reduce by half the expected time to achieve the initial innovations. Decreasing marginal productivity (in the single input) is the standard assumption in the microeconomic theory of patent races.

[2] A related task is present in a work by Zeira (2003), whose interest is in the choice of innovators between simple innovations and more difficult but radical innovations and across multiple research strategies, and especially in Denicolò - Zanchettin (2006).

[3] See Minnitti (2007) for related results.

[4] Blundell, Griffith - Van Reenen (1999) provide wide evidence based on British manufacturing which is «*in line with models where high market share firms have greater incentives to preemptively innovate*». In a similar vein, Nicholas (2003) presents an historical analysis on 1920s America and shows that «*firms with high levels of market power tended to innovate more because they had strong incentives to do so pre-emptively*». Ogilvie (2004a,b) provides an hystorical perspective on the relationship between barriers to entry, lack of innovation by leaders and lack of growth.

while in the more realistic case of decreasing marginal productivity, they allow entry but still invest more than any other firm, which implies partial persistence of monopolistic positions[5]. In this case, I show that the engine of growth is more powerful, in the sense that the value of innovations is enhanced, the aggregate incentives to invest are increased and this speeds up the growth process. This characterization of growth driven by dominant firms may help explain the persistence of technological leadership in the US, whose large corporations tend to perpetuate their leadership in most high-tech sectors and that of US in the world economy. A contribution of this paper is in the development of dynamic programming techniques to solve analytically a complex general equilibrium model with Schumpeterian growth and endogenous persistence of leadership[6].

The framework developed in this article allows to explore how different factors and policies can affect the engine of growth. In particular, here I will augment the model with another generic source of growth, which may just be a traditional exogenous technological progress or it may be microfounded in some endogenous way. This allows to show a surprising result: an increase in growth due to other sources may reduce the incentives to innovate. Consequently, even if innovation is the main engine of growth (in the sense that it actually contributes to most of the growth rate), growth and investment in innovation may be negatively correlated (over time or across countries). This result is due to the increase in the interest rate associated with higher growth which may crowd out some firms from the innovation sector: this suggests that empirical tests of Schumpeterian growth theories should separate the direct positive effect of innovation on growth from the feedback effect of growth on innovation.

Finally, following the new-keynesian tradition, and in particular a recent contribution by Barro and Tenreyro (2006), I extend the model to a monetary economy drawing some new implications on the relationship between inflation and endogenous growth. Price stickiness induces an inverted-U relation between inflation and long run growth which is broadly consistent with available empirical evidence: excessive inflation erodes the monopolistic profits of innovators reducing the aggregate incentives to invest and hence reducing growth.

In what follows, Section 2 describes the model and Section 3 solves for the equilibrium organization of the R&D sector and for the general equilibrium. Section 4 derives the optimal R&D policy and Section 5 endogenizes the persistence of monopolies. Section 6 discusses a few extensions, while Section 7 concludes. Technical details are left to the Appendix.

[5] In a recent important paper, Segerstrom (2007) has developed a model where incumbent monopolists invest in R&D because they can use a different innovation technology than the one adopted by the other firms. This approach basically assumes cost advantages in the innovation activity for the monopolists and allows to study their persistence, but it does not explain its ultimate source. For related dicussions see also Grieben (2005), Denicolò and Zanchettin (2006) - Impullitti (2006a, b).

[6] In Etro (2004) I could not solve such a problem and had to rely on numerical simulations. The current study corrects the imprecisions derived within that early investigations.

II - THE MODEL

Let us consider an infinite horizon representative agent with isoelastic utility:

$$U = \int_0^\infty \frac{C_t^{1-\gamma} - 1}{1-\gamma} e^{-\rho t} dt \quad \text{with} \quad \gamma > 0 \tag{1}$$

where $\rho > 0$ is the time preference rate. The agent can consume its income or save it and finance R&D activities providing a rate of return r[7]. This implies that consumption grows at the rate:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma} \tag{2}$$

Output Y can be used for consumption C, production of intermediate goods X or investment in R&D and it is produced according to the constant return to scale function introduced by Barro and Sala-i-Martin (1995, Ch. 7):

$$Y = AL^{1-\alpha} \sum_{j=1}^{N} (q^{\kappa_j} X_j)^\alpha \tag{3}$$

where A is Total Factor Productivity, L is the fixed labor force, X_j is the intermediate good j of quality k_j, N is the constant number of intermediate goods, $q > 1$ and $\alpha \in (0, 1)$. The advantage of the Barro and Sala-i-Martin framework is that technological progress due to innovations realizes independently for each intermediate good: hence, it allows to think of small and frequent innovations rather than innovations for general purpose technologies.

2.1. *The output and inputs markets*

The markets for the final good, which is the *numeraire*, for labour and for credit (to firms investing in R&D) are perfectly competitive. Each intermediate good is monopolistically produced by a single firm with a patent on it. I will define $1 + \mu$ as the optimal price for this monopolist (so that μ is the mark up on the unitary marginal cost), which may be microfounded as the monopolistic price $1/\alpha$ for drastic innovations, the limit price q for non drastic ones or in other ways (even taking into account other factors, like taxation). The aggregate quantity produced of intermediate good j can be determined as:

$$X(\kappa_j) = \left(\frac{\alpha A}{1+\mu}\right)^{1/(1-\alpha)} L q^{\kappa_j \alpha/(1-\alpha)}. \tag{4}$$

[7] Investment takes place immediately since there is not storage in this economy. One could extend the model with investment goods and depreciation.

Substituting the quantity $X(\kappa_j)$ from (), we obtain the output of final goods:

$$Y = A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{1+\mu}\right)^{\frac{\alpha}{1-\alpha}} LQ \tag{5}$$

and total amount of intermediate goods $X = [\alpha A/(1+\mu)]^{\frac{1}{1-\alpha}}LQ$, where we have introduced the Barro and Sala-i-Martin aggregate quality index $Q \equiv \sum_{j=1}^{N} q^{\frac{\kappa_j \alpha}{1-\alpha}}$. The flow of profit for each intermediate good producer with the sector's highest quality rung κ_j is $\pi(\kappa_j) = \mu X(\kappa_j)$.

2.2. *The market for innovations*

Every time an innovation is obtained for some intermediate good j, the innovator starts producing it with the cutting-edge technology and new firms begin to invest to find out the next innovation. I assume that R&D requires a monetary investment, but the Aghion - Howitt (1992) approach using labour as the factor of production in the R&D sector would not alter the qualitative results. To participate in the competition for the innovation, or shortly the patent race, each firm i has to pay a fixed cost $F(\kappa_j)$ (which may include an entry fee set by the government) and spend a flow of resources $z_i(\kappa_j)$. A constant subsidy rate could be introduced as in Etro (2006d).

Contrary to the usual literature, which assumes constant marginal productivity - equivalent to constant returns to scale since there is just one input - in the R&D sector, I will introduce decreasing marginal productivity in the production of new ideas. In particular the investment for firm i gives birth to the innovation k_j according to a Poisson process with arrival rate $h_i(k_j)$ given by a concave function of $z_i(\kappa_j)$. To obtain closed form solutions, I assume the following specification:

$$h_i(k_j) = [\phi(\kappa_j)z_i(\kappa_j)]^{\epsilon} \tag{6}$$

where the function $\phi(k_j)$ expresses how difficult is to discover technology k_j and $\epsilon \in (0,1]$ represents the degree of returns to scale in the innovation sectors or the elasticity of expected revenue with respect to the flow of investment. This parameter is unitary in the existent versions of the quality-ladder model (starting with Aghion and Howitt, 1992, 1998), but empirical research, for instance by Cohen and Klepper (1996) and Kortum (1993) suggests an elasticity much smaller than 1[8]. The arrival rate of innovation κ_j, will just be the sum of the individual arrival rates of the $n(k_j)$

[8] Kortum (1993) suggests a range between 0.1 and 0.6 for this elasticity. Segerstrom (2007) assumes that decreasing returns hold just for the incumbent monopolist, while constant returns to scale characterize all the other firms. He solves the model through simulations and assumes $\epsilon = 0.3$ as the average between the values proposed by Kortum.

entrants[9] plus the one of the incumbent, indexed with M:

$$p(\kappa_j) = \sum_{i=1}^{n(k_j)} [\phi(\kappa_j) z_i(\kappa_j)]^{\epsilon} + [\phi(\kappa_j) z_M(\kappa_j)]^{\epsilon} \tag{7}$$

Hence, there are decreasing returns both at the firm level and at the industry level, as suggested by the empirical evidence in this field. Using the properties of Poisson processes in a standard fashion, the expected net profit of entrant i in the patent race in sector j when the current quality is k_j can be written as:

$$\Pi^i(k_j) = \frac{[\phi(\kappa_j) z_i(\kappa_j)]^{\epsilon} \mathbf{V}(\kappa_j + 1) - z_i(\kappa_j)}{r + p(\kappa_j)} - F(\kappa_j) \tag{8}$$

where $r + p(\kappa_j)$ could be defined as the effective discount factor at the time of the innovation of vintage κ_j and $\mathbf{V}(\kappa_j + 1)$ is the value of being monopolist with the next technology $k_j + 1$. Since also the incumbent monopolist with the technology k_j can invest to innovate, we need to consider its objective function, which is given by the Bellman equation:

$$\mathbf{V}(\kappa_j) = \max_{z_M \geq 0} \left\{ \frac{[\phi(\kappa_j) z_M]^{\epsilon} \mathbf{V}(\kappa_j + 1) + \pi(\kappa_j) - z_M}{r + p(\kappa_j)} - F(\kappa_j) \right\} \tag{9}$$

where the fixed cost is paid only if $z_M > 0$. Deep down, this value of the innovation is the engine of growth, because what drives investment and growth is exactly the attempt to conquer this value. In the standard literature, monopolists do not invest, hence the value of leadership is just the expected profits from the next innovation. As we will see later on, monopolists invest when they have a leadership and in that case, the value of the innovation includes also the option value of a persistent leadership, which fundamentally modifies the incentives to invest.

To close the model we will need some assumptions on the functional forms for $\phi(\kappa_j)$ and $F(\kappa_j)$. We assume that new ideas are more difficult to obtain when the scale of the sector, as represented by expected production with the new technology, increases, and that the fixed cost is a constant fraction of the expected cost of production with the new technology:

$$\phi(\kappa_j) = X(\kappa_j + 1)^{-1}, \quad F(\kappa_j) = \frac{\eta X(\kappa_j + 1)}{[r + p(\kappa_j + 1)]} \tag{10}$$

with $\eta \in (0, \mu)$ to make the problem interesting (otherwise there would not be any research). I want to capture the idea that the larger is the scale of expected production of a firm, the larger are the costs necessary to discover it, to develop the associated technology and the infrastructures needed to adopt this technology (think of new assembly lines, training of workers, construction of prototypes and samples). These assumptions will deliver a balanced growth path and will avoid scale effects

[9] In all the paper we will consider the number of firms as a real number greater than unity.

on the equilibrium growth rate, which is in line with the Jones's critique to the first generation of quality-ladder models[10].

Given this set-up, one can make different assumptions on the kind of competition in the market for innovations. In this paper I will consider the traditional Nash case and the Stackelberg case (in which the current monopolist has a first mover advantage) with free entry.

III - THE NUMBER AND SIZE OF R&D INVESTORS

In this section I model competition in the market for innovation in the Nash fashion. As a consequence of the Arrow paradox, under free entry the leader does not invest in R&D, because its best strategy is to stay out from the patent race and enjoy the profits from its current patent until a new innovation will make it obsolete[11]. Competition for innovations is just between outsiders and the scope of this section is to characterize the equilibrium organization of the R&D sector, the number of firms investing and their size together with the usual macroeconomic variables as aggregate growth.

The lack of investment by monopolists implies that the value of being a monopolist with technology k_j, (9), boils down to:

$$\mathbf{V}(\kappa_j) = \frac{\pi(\kappa_j)}{r + p(k_j)} = \frac{\mu\left(\dfrac{\alpha q^{\kappa_j \alpha} A}{1+\mu}\right)^{\frac{1}{1-\alpha}} L}{r + p(k_j)} \tag{11}$$

Each firm chooses its investment in R&D $z_i(\kappa_j)$ to maximize (8) taking (7) into account and taking the strategies of the other firms, the value of the next innovation and the interest rate as given, while the free entry condition sets the expected profits (8) equal to zero providing the equilibrium number of entrants $n(\kappa_j)$. This implies the investment per firm:

$$z(\kappa_j) = \epsilon^{\frac{1}{1-\epsilon}} \phi(\kappa_j)^{\frac{\epsilon}{1-\epsilon}} \left[\mathbf{V}(\kappa_j + 1) - F(\kappa_j)\right]^{\frac{1}{1-\epsilon}} \tag{12}$$

which is increasing in the value of the innovation net of the fixed cost of entry, while is independent from the interest rate.

Substituting the endogenous value of innovation (11) and using our assumptions

[10] See Jones (1995) and Barro - Sala-i-Martin (2004).

[11] See for instance Etro (2004) for a proof in this same context. The general intuition for this result is simple. While the gains from an innovation for the incumbent monopolist are just the differential between profits obtained with the next patent and those obtained with the current one, the gains for any outsiders are the full profits from the next patent. Hence the incumbent has lower incentives to invest in R&D. The expected gains of the incumbent are even diminished when the investment of the outsiders increases. And when the latter arrives to the point that expected profits for the outsiders are zero, the incumbent has no more incentives at all to participate to the patent race.

(10) in the expression for investment and in the free entry condition, we can explicitly obtain the investment by each firm and the number of firms in this particular patent race[12]. It turns out that the former is increasing in the quality achieved in the single sector, since this implies higher demand and hence higher expected profits for the corresponding intermediate product, while the number of firms and the aggregate probability of innovation in each patent race turn out to be the same in all sectors. To see this, it is easy to use the free entry condition setting (8) equal to zero to express the effective discount rate. Using this to explicit the expected value of innovation (11)[13], and substituting in (12) we can obtain the equilibrium flow of investment per firm:

$$z(\kappa_j) = \frac{\epsilon\eta(\mu-\eta)}{[\mu-\epsilon(\mu-\eta)]}\left(\frac{\alpha A}{1+\mu}\right)^{\frac{1}{1-\alpha}} Lq^{\frac{\alpha}{1-\alpha}(\kappa_j+1)} \tag{13}$$

which is strictly positive. This investment is proportional to the future scale of production and increasing in the degree of returns to scale, ϵ, since this makes investment more productive. For a given scale of production, it is increasing in the mark up μ, which is exactly the core of the Schumpeterian idea that monopolistic profits drive innovation of single firms[14]. The effect of higher fixed costs on investment can be shown to be non monotonic, positive for η low but negative for η high enough: on one side high fixed costs reduce expected profits for a given life of the patent, but on the other, they reduce the innovation rate in the future so as to increase the expected life of the patent.

Defining simply $p = p(k_j)$ the aggregate arrival rate of innovations in every sector at each point in time, we can now derive from (5) the growth rate of income which corresponds to the expected growth rate of the aggregate quality index Q:

$$E\left(\frac{\dot{Q}}{Q}\right) = g = \frac{p\alpha \ln q}{1-\alpha} \tag{14}$$

It is standard to verify that on a balanced growth path consumption and income must grow at the same rate, which allows to endogenize the interest rate and consequently to fully characterize all the other equilibrium variables, that is the growth

[12] In the Barro - Sala-i-Martin (1995) model, the arbitrage equation for the patent race k_j pins down the investment in innovation in the patent race $k_j + 1$ without a clear economic intuition. Instead, in our model, the free entry condition for the patent race k_j pins down the number of firms investing in innovation in the patent race k_j and, together with their profit maximising choices, their individual investments in the same patent race.

[13] Notice that the value function of the monopolistic position grows at a constant rate $\mathbf{V}(\kappa_j+1) = \mathbf{V}(\kappa_j)q^{\frac{\alpha}{1-\alpha}}$.

[14] When the mark up is endogenously given by $\mu = \left(\frac{1-\alpha}{\alpha}\right)$, the flow of investment is ambiguously affected by the factor share of the intermediate products α (or by their related elasticity of demand, $1/(1-\alpha)$): on one side, the higher is α, the lower the profit margin, which reduces the incentives to invest, on the other side, the higher is α, the higher will be future production and the associated profits, which promotes investment.

rate, the arrival rate of innovations and the number of firms in the R&D sector as functions of behavioral, technological and policy variables. In particular, we have: :

Proposition 1: Under Nash competition in the market for innovations, the equilibrium growth rate is:

$$g = \frac{[\epsilon(\mu-\eta)]^{\epsilon}\left[\dfrac{\mu(1-\epsilon)+\epsilon\eta}{\eta}\right]^{1-\epsilon} - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{15}$$

which is decreasing in η, while it is increasing in μ.

As one could expect, the more costly are innovations, the lower is equilibrium growth; increases in the size of the innovations (for instance due to some general purpose technology which enhances technological progress) or in monopolistic mark-ups promote growth, while it can be shown that the relation between growth and ϵ is U-shaped.

The equilibrium arrival rate of innovations is directly proportional to the above growth rate, while the equilibrium number of firms is:

$$n = \frac{\left[\dfrac{\mu-\epsilon(\mu-\eta)}{\eta}\right] - \rho\left[\dfrac{\mu-\epsilon(\mu-\eta)}{\epsilon\eta(\mu-\eta)}\right]^{\epsilon}}{1+\alpha\gamma\ln q/(1-\alpha)} \tag{16}$$

hence higher size innovations are associated with higher growth but fewer firms (and less frequent innovations). Contrary to standard models without scale effects, this one implies that R&D policy, in this case R&D subsidization, has a positive effect not only on investment in innovation at the firm level and at the aggregate level, but also on the growth rate.

In conclusion, we have extended the Barro and Sala-i-Martin framework to explicit characterization of realistic patent races. In that framework, the organization of the R&D sector is indeterminate[15], while here it can be accurately described and used for welfare analysis, which is the focus of the next section.

IV - OPTIMAL R&D POLICY

I will now derive the optimal organization of the R&D sector and the optimal R&D policy. I will provide an heuristic solution for the social planner problem with the purpose to obtain a more intuitive presentation; an equivalent proof can be obtained using standard optimal control arguments.

[15] Notice that approaching constant returns to scale in our model (that is when $\epsilon \to 1$ and $\eta \to 0$), the investment by each firm and the number of firms become indeterminate, but the equilibrium growth rate converges to the traditional one (see Barro and Sala-i-Martin, 2004): $g \to (\mu-\rho)/[\gamma+(1-\alpha)/\alpha \ln q]$.

First of all, it is immediate to derive from the concavity of the arrival rate that it is optimal to allocate equal flows of investment between all the R&D laboratories (and in this respect the decentralized equilibrium does the correct thing). Now, we guess that these flows are linear functions of the future scale of production, let us say $z(\kappa_j) = \beta X(\kappa_j + 1) = \beta q^{\frac{\alpha}{1-\alpha}} X(\kappa_j)$ for each firm in sector j, where β is a parameter to choose optimally. Let us keep the production of intermediates at the level chosen by the monopolist in the decentralized equilibrium. As well known, a social planner would not distort the choice of the input mix, hence we are basically solving for a second best allocation[16].

The resource constraint of the economy must take into account the fixed costs, which are paid only at the beginning of each new patent race. Without loss of generality let us assume that the economy devotes a flow of resources for this purpose in each sector[17]. If the number of sectors N is high enough, one can approximate this flows, say $f_j(\kappa_j)$ with those equating their expected present value $f_j(\kappa_j)/\left[r + p(k_j)\right]$ to the fixed cost $F(\kappa_j)$, that is with $f_j(\kappa_j) = \eta X(\kappa_j + 1)$. Using the (4) and (5), we can rewrite the resource constraint as:

$$Y = \frac{X}{\alpha} = C + \sum_{j=1}^{N} X_j(\kappa_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} z_i(\kappa_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} f_j(\kappa_j) =$$

$$= C + X + n(\beta + \eta) q^{\frac{\alpha}{1-\alpha}} X$$

from which we derive an expression for consumption holding at each point in time. Under the optimal allocation of resources, growth is determined by the rate of innovation as:

$$g = n\left[\phi(\kappa_j) z(\kappa_j)\right]^{\epsilon} \frac{\alpha}{1-\alpha} \ln q = \beta^{\epsilon} \frac{\alpha n}{1-\alpha} \ln q \tag{17}$$

Given a constant growth rate of consumption, intertemporal utility is finite as long as $\rho > (1-\gamma)g$, and can be written as:

$$U = \int_0^{\infty} \frac{C_t^{1-\gamma}}{1-\gamma} e^{-\rho t} dt = \frac{C_0^{1-\gamma}}{(1-\gamma)[\rho - (1-\gamma)g]} \tag{18}$$

Finally, substituting initial consumption and the expression for growth in (18), we can summarize the social planner problem as:

$$\max_{n,\beta} \frac{X_0{}^{1-\gamma}\left[\frac{1-\alpha}{\alpha} - n(\beta+\eta) q^{\frac{\alpha}{1-\alpha}}\right]^{1-\gamma}}{(1-\gamma)\left[\rho - (1-\gamma)\beta^{\epsilon} \frac{\alpha n}{1-\alpha} \ln q\right]} \tag{19}$$

[16] The first best allocation would be obtained by subsidizing monopolists in such a way that their price equates marginal cost. This theorical solution to monopolistic distortions is quite unrealistic.

[17] We may think of a perfectly competitive banking sector specialized in venture-capital financing. Banks finance the fixed cost for the investment in a new technology and investors commit to a flow of payment until the new technology is obtained.

which puts in clear evidence the basic trade-offs. A higher number of firms or a higher flow of investment per firm imply a higher growth rate of consumption but with a lower initial consumption level (and the time preference rate and the elasticity of substitution govern this trade-off in a standard fashion), but the weights on benefits and costs are different for the two choice variables. The higher is the fixed cost parameter η the more costly is to increase the number of firms rather than the flow of investment. Finally, also the size of the innovations q and the parameter α characterizing the elasticity of demand of intermediate goods influence the trade-off. If an interior solution exists, the first order conditions for the social planner problem (19) with respect to β and n are:

$$q^{\frac{\alpha}{1-\alpha}}\left[\rho-\beta^{\epsilon}\frac{(1-\gamma)\alpha n\ln q}{1-\alpha}\right]=\frac{\epsilon\alpha\beta^{\epsilon-1}\ln q}{1-\alpha}\left[\frac{1-\alpha}{\alpha}-n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]$$

$$(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\left[\rho-\beta^{\epsilon}\frac{(1-\gamma)\alpha n\ln q}{1-\alpha}\right]=\beta^{\epsilon}\frac{\alpha\ln q}{1-\alpha}\left[\frac{1-\alpha}{\alpha}-n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]$$

Dividing one by the other we obtain $\beta^{*}=\epsilon\eta/(1-\epsilon)$, which implies the optimal flow of investment in R&D per firm:

$$z^{*}(\kappa_j)=\frac{\epsilon\eta}{1-\epsilon}\left(\frac{\alpha A}{1+\mu}\right)^{\frac{1}{1-\alpha}}Lq^{\frac{\alpha}{1-\alpha}(\kappa_j+1)} \tag{20}$$

Let us compare (20) with the equilibrium flow of investment (13) for a given level of expected production. While the equilibrium investment of each firm increases in the profit margin μ, the optimal investment is independent from that and it is positively correlated with the elasticity of expected revenue with respect to the flow of investment. Moreover, it is immediate to verify that the equilibrium investment is always below the optimal level: the decentralized equilibrium with Nash competition in the market for innovations always implies a sub-optimal flow of investment in R&D per firm.

In other words, the organization of the R&D activity is biased toward too small firms. Clearly this does not mean that there is not enough R&D activity, because there may be too many small firms, but just that the division of total investment is inefficient. It would be efficient to increase the investment of each single firm in R&D. When growth is led by technological progress, a country with an industrial structure characterized by small firms achieves inefficient results, and in particular it could grow more (or enjoy a larger welfare) if its firms were increasing in size.

This general conclusion may shed new light on the problems of countries that do not grow much, are characterized by many small firms investing too little in R&D and lack large and innovative corporations. For instance, this is the case of Italy, whose industrial structure is characterized by a large number of small and medium firms whose innovative capacity is quite limited and whose focus has gradually moved away from high-tech sectors. The reasons for the lack of growth in the size of Italian firms have been usually associated with the family based structure of Italian capitalism or with credit rationing problems, but the endogenous tendency toward small innovative firms suggested here may be part of the story (since R&D subsidization has been always limited in Italy compared to other western countries). Not by chance, the Italian endogenous response to this problem (without proper equivalents around the world) has been the delegation of innovative activities to *industrial districts*, that is organizations of more firms investing in the same sector on a larger scale. Notice, however, that the lack of large corporations in high-tech sectors is also a problem in other European countries compared to the US[18].

Notice that our analysis of the market for innovations and of the incentives of firms to invest in R&D is based on expected values of projects and on their expected costs, as if investors were risk neutral or were able to perfectly diversify their portfolio of R&D projects (with small projects and idiosyncratic uncertainty). It would be interesting to consider the role of risk diversification in the investment choices. Some observers think that the dimensional problem of Italian firms mentioned above generates inefficiency also because it does not allow enough risk diversification in R&D projects at the firm level, thus lowering the aggregate amount of resources devoted to R&D[19].

Let us now look at the number of firms investing in R&D, which allows us to obtain a complete view on the organization of R&D investment. From the first order conditions we obtain the optimal number of R&D laboratories as:

$$n^* = \frac{1-\alpha}{\gamma\alpha\epsilon^\epsilon}\left[\frac{1-\epsilon}{\eta q^{\frac{\alpha}{1-\alpha}}} - \frac{\rho}{\ln q}\left(\frac{1-\epsilon}{\eta}\right)^\epsilon\right] \tag{21}$$

which is decreasing in ϵ at least for ϵ high enough: this implies that when the marginal productivity of the investment is high enough, it is optimal to have just one laboratory investing in R&D. However, let us focus on the case where the optimal number of laboratories is larger than one. Comparing (21) with its equilibrium counterpart (16), it can be verified that the decentralized equilibrium implies too few firms for any γ smaller than a cut-off. This result has a simple intuition: when γ is low, it is optimal to choose a high growth rate of consumption, hence the social value of innovations, which is what drives growth, is high. On the other side, the private value of innovations depends on market features which are independent from consumers preferences (except for an indirect channel going through the interest rate). Hence, for low enough γ, the social value of innovations is larger enough than the private value and the optimal number of firms becomes larger than the equilibrium number.

Finally, substituting β^* and n^* in our expression for growth (17), we obtain that *the optimal growth rate is:*

$$g^* = \frac{1}{\gamma}\left[\epsilon^\epsilon\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left[1-q^{-\frac{\alpha}{1-\alpha}}\right]\left(\frac{1-\alpha}{\alpha}\right)-\rho\right] \tag{22}$$

[18] For recent policy analysis on the benefits of market reforms on growth taking in considerations the market for innovations, see Faini et al. (2006). For other related empirical investigations see Chaudhuri - Flamm (2005), Johansen - Damm (2005) and Etro (2006b).

[19] I am thankful to a referee for pointing this out.

which decreases with η and is higher than the equilibrium growth rate any γ smaller than a cut-off.

The optimal growth rate clearly reflects the trade-off between the number of laboratories and individual investment. From a policy point of view, the optimal R&D policy should try to implement the optimal organization of the R&D sector, that is to achieve the optimal number of firms and the optimal investment per firm. The optimal R&D policy requires two policy tools, an R&D subsidy and an entry fee that are derived in Etro (2006d).

V - GROWTH DRIVEN BY MARKET LEADERS

An important stylized fact about innovations is that many of them are due to incumbent monopolists and that a lot of the investment in R&D is actually done by both incumbents and new firms. One of the industry leaders investing more in innovation is Microsoft, the leading firm in operating systems: in 2000, its expenditure in R&D was 3700 millions $, corresponding to 16.4% of its total sales. In the following years Microsoft continued to invest at these levels and after almost two decades of leadership, it still keeps being one of the most successful and innovative companies in the world, whose innovations have contributed substantially to the global progress. High investments can also be found in many other major firms of high tech sectors. In the same year, the R&D/Sales ratio was 15% for Pfizer and 5.8% for Merck, two leaders in the pharmaceutical sector, 11.5% for Intel, leader in the chips market and 5.8% for IBM, and 5.4% for Hewlett Packard, two leaders in computer technologies and services, 11.8% for Motorola and 8.5% for Nokia, leaders in wireless, broadband and automotive communications technologies, 10% for Johnson & Johnson, the world's most comprehensive manufacturer of health care products and services, 6.6% for 3M and 6.3% for Du Pont, which are active in many fields with a leading role, 5.6% for Xerox (mostly focused on the legendary Palo Alto Research Center) and for Kodak, leaders in the markets for printers and photographs.

Existing models about innovation and growth are inconsistent with this overwhelming evidence, since under Nash competition and free entry, as we have seen also in the previous sections, an incumbent monopolist has no incentives to invest in R&D because of the Arrow (1962) effect: its incentives derive from the differential of profits between the next and the current innovation, while those of the outsiders derive from the all profits from the new innovation and are higher, hence under free entry just outsiders invest.

In Etro (2004), I have rationalized investment of the incumbents in a partial equilibrium framework showing that monopolists invest in R&D more than any other firm as long as they are leaders in the patent race and entry is free in the same. This behavior of the leaders under free entry is a particular case of a much more general result established in Etro (2002) where I have shown that Stackelberg leaders are always aggressive (under quantity or price competition or in patent races as here) whenever the number of followers is endogenous. In our context, the requirement that incumbents are leaders in the market for innovations is realistic: after all, it is reasonable to imagine that they have a credible commitment to invest a certain amount of resources in R&D. Otherwise, we can imagine that the incumbent monopolist can undertake some preliminary investment which affects its profitability from engaging in R&D activity (like building laboratories, hiring researchers or borrowing to invest). As I have shown in Etro (2006a,b), these strategic investments allow to reproduce similar outcomes to Stackelberg equilibria with free entry: leaders always overinvest strategically to be able to be aggressive in the market[20].

Let us consider the market for innovation described in Section 2, where (entrant) and (9) are the objective functions of the entrants and the leader. In this set up, the partial equilibrium for each patent race is simple to derive (Etro, 2004). When marginal productivity is close to constant (for high ϵ), it is optimal for the monopolist to deter entry investing just enough in R&D to make unprofitable for any follower to engage in R&D activities, and this delivers complete persistence of monopolies[21]. This case is examined in the Appendix, while here I will focus on the more realistic case where entry by outsiders takes place and the persistence of monopolies is only partial. This requires that investment in R&D faces significant decreasing marginal productivity, i.e. ϵ is small enough[22].

When the free entry condition pins down the number of followers in each sector, it is easy to verify that their optimal strategy $z(k_j)$ is always independent from the one of the leader (while the number of followers decreases in the investment of the leader), hence the effective discount rate $r + p(k_j)$ must be also independent from the leader's strategy (Etro, 2004). The leader chooses its investment $z_M(k_j)$ to solve the problem (9), where the effective discount rate is independent from its choice and hence taken as given. From the corresponding first order condition, the one of the followers and the zero profit condition on the profits of the followers, we obtain:

$$z(\kappa_j) = \frac{\left\{\epsilon\phi(\kappa_j)\left[\mathbf{V}(\kappa_j+1) - F(\kappa_j)\right]\right\}^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)} < z_M(\kappa_j) = \frac{\left[\epsilon\phi(\kappa_j)\mathbf{V}(\kappa_j+1)\right]^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)} \quad (23)$$

Notice that the investment of each follower is increasing in the value of innovat-

[20] On the market leaders approach see Wiethaus (2005a, b), Zigic et al. (2006), Cozzi (2007), Czarnitzki - Kornelius (2007) and Reksulak et al. (2006). For a survey and a discussion of the antitrust implications see Etro (2006b, c; 2007).

[21] I am thankful to Robert Barro for leading me to realize this fact. In a related research, he has obtained a similar result considering a different Poisson process generating innovations (see Barro - Sala-i-Martin, 1995, Exercise 7.4) with a concave function at the aggregate level, $h(Z)$, where Z is the total investment in innovation. Barro assumes that each firm investing z discovers the new technology with instantaneous probability $\frac{z}{Z}h(Z)$, where $Z = \sum z$, so that there are (approximately) constant returns to scale at the firm level but decreasing returns at the aggregate level. Not surprisingly, the Barro's model with Stackelberg competition implies that only the leader invests in innovation to preempt entry by outsiders.

[22] Segerstrom (2007) has criticized this approach for implying a low persistence of monopolies. In reality, this approach is also consistent with complete persistence (for ϵ high enough). In a realistic set up, however, monopolies are persistent, but not eternal.

ing net of the fixed cost, while the investment of the leader is independent from the fixed cost. The solution for the equilibrium is complicated from the fact that now we do not know what is the value of being a monopolist, since this is the solution to a Bellman equation. But this value is what drives the incentives to invest in innovation, that is the engine of growth.

This problem can be solved using standard dynamic programming methods[23]. A contribution of this work is also in providing a way to solve this kind of problems, which are likely to emerge whenever one is dealing with Schumpeterian models of growth where incumbent monopolists engage in R&D activity together with outsiders (see also Segerstrom, 2007). Let us look for a balanced growth path with constant values for the growth rate, the interest rate, the arrival rate of innovations and the number of firms investing in patent races.

The two equilibrium equations expressing the growth rate of income and consumption are the same as before and provide a common growth rate directly related to the effective discount rate:

$$g = \frac{r + p - \rho}{\gamma + (1 - \alpha)/\alpha \ln q} \tag{24}$$

To derive the equilibrium values for the value function $\mathbf{V}(\kappa_j)$, the functions $z(k_j)$ and $z_M(k_j)$ and the equilibrium values for g, r, p and n, we can adopt the method of undetermined coefficients. Let us guess a functional form for the value function as:

$$V(\kappa_j) = V(\kappa_j - 1) q^{\frac{\alpha}{1-\alpha}} = \psi \frac{X(\kappa_j)}{r + p} \tag{25}$$

where ψ is a coefficient to be determined which can be interpreted as the rate of return from leadership. It must be larger than μ, otherwise the value of being a leader investing in R&D would be smaller than the value of being a leader without investing, or in other words it would be optimal to stay out of the patent race for the leader. Using this functional form in our expressions (23) we have:

$$z(\kappa_j) = \left(\frac{\epsilon(\psi - \eta)}{r + p}\right)^{\frac{1}{1-\epsilon}} q^{\frac{\alpha}{1-\alpha}} X(\kappa_j), \quad \mathrm{z_M(\kappa_j) = z(\kappa_j)}\left(\frac{\psi}{(\psi - \eta)}\right)^{\frac{1}{1-\epsilon}} \tag{26}$$

Then, substituting in the Bellman equation we obtain:

$$\mathbf{V}(\kappa_j) = \frac{(\phi z_M)^{\epsilon} \mathbf{V}(\kappa_j + 1) + \pi(\kappa_j) - z_M}{(r + p)} - F(\kappa_j) =$$

$$= \frac{X(\kappa_j)}{r + p} \left[\mu + \epsilon^{\frac{\epsilon}{1-\epsilon}} \left(\frac{\psi}{r + p}\right)^{\frac{1}{1-\epsilon}} q^{\frac{\alpha}{1-\alpha}} (1 - \epsilon) - \eta q^{\frac{\alpha}{1-\alpha}}\right]$$

whose right hand side contains the sum of the mark up from the current innovation

[23] See Stokey - Lucas with Prescott (1989) and Ljungqvist - Sargent (2004).

and a second term which represents the option value of being the leader and having the opportunity to remain the dominant firm in the future innovation: this option has a positive value which directly comes from the leadership. Using (25) and solving the Bellman equation for the effective discount rate we have:

$$r + p = \epsilon^{\epsilon}\left[\frac{(1-\epsilon)q^{\frac{\alpha}{1-\alpha}}}{\psi - \mu + \eta q^{\frac{\alpha}{1-\alpha}}}\right]^{1-\epsilon}\psi \tag{27}$$

which provides a negative relation between the effective discount rate $r + p$ and the rate of return from leadership ψ (for ψ small enough): the higher is the effective discount rate, the shorter is the lifetime of an innovation, and hence the lower is the value from being a leader.

Moreover, the zero profit condition for the followers provides another expression for the effective discount rate which is analogous to the one derived in the case of Nash competition:

$$r + p = [\epsilon(\psi - \eta)]^{\epsilon}\left[\frac{\psi(1-\epsilon) + \epsilon\eta}{\eta}\right]^{1-\epsilon} \tag{28}$$

This is a positive relation between the effective discount rate $r + p$ and the rate of return from leadership ψ: the higher is the value of being a leader, the larger will be the investment in R&D and hence the probability of innovation and the effective discount rate.

Equating (27) and (28) we obtain the equilibrium value for ψ which provides all the equilibrium relations. An implicit expression for ψ is given by:

$$\psi = \mu + \frac{(1-\epsilon)\eta q^{\frac{\alpha}{1-\alpha}}\psi^{\frac{1}{1-\epsilon}}}{(\psi - \eta)^{\frac{\epsilon}{1-\epsilon}}[\psi(1-\epsilon) + \epsilon\eta]} - \eta q^{\frac{\alpha}{1-\alpha}} > \mu \tag{29}$$

An immediate conclusion derives from the fact that $\psi > \mu$:[24] the effective discount rate and hence both the aggregate probability of innovation and the growth rate must be higher than under Nash competition[25]. The incumbency advantage adds a turbo to the engine of growth because it endogenously increases the value of innovations associating with them an option to persistent leadership, which increases aggregate investment and hence growth:

[24] When $\alpha = \epsilon = 0.5$, this reduces to $\psi = \mu + q\eta^3/(\psi^2 - \eta^2)$. Then, assuming $q = 1.1$ and $\eta = 0.1$, a mark up of 20% implies a return from leadership of about 23%. Then the equilibrium growth rate without incumbency advantages would be 1.58%, the one with growth driven by monopolists 1.92%, even if the optimal growth rate would be 8%.

[25] This does not need to be the case when incumbents find optimal to deter entry (that is for high enough ϵ). In that case, their entry deterrence investment in R&D may be below the equilibrium investment under Nash competition in the patent races, and consequently growth driven exclusively by monopolists may be lower (as in the model with exogenous investment by the monopolists in Barro - Sala-i-Martin, 1995).

Proposition 2: Under leadership by incumbent monopolists in the R&D sector, the return from leadership is higher than under pure Nash competition because of the option value of monopoly persistence.

Moreover, we can easily verify that both the return from leadership ψ and the effective discount rate and hence the growth rate are increasing in the mark up μ. An increase in the fixed cost of innovation through η decreases the effective discount rate and hence the growth rate of the economy, but it has ambiguous effects on the value of being a leader. In conclusion, we have:

Proposition 3: For ϵ small enough, under Stackelberg competition in the market for innovations, monopolists invest in R&D more than any outsider and the equilibrium growth rate is:

$$g = \frac{\psi \epsilon^{\epsilon} \left(\frac{1-\epsilon}{\eta} \right)^{1-\epsilon} \left[\frac{\eta q^{\frac{\alpha}{1-\alpha}}}{\psi - \mu + \eta q^{\frac{\alpha}{1-\alpha}}} \right]^{1-\epsilon} - \rho}{\gamma + (1-\alpha)/\alpha \ln q} \tag{30}$$

where ψ is given by (29), and it is decreasing in η and increasing in μ. Moreover, growth is higher than under pure Nash competition.

Clearly, when the engine of growth is given by persistent monopolistic positions as in this model, the investment by each firm increases. It can be shown that the optimal allocation of resource can be achieved with a positive subsidy for the entrants, a smaller one for the incumbent monopolists and an appropriate entry fee to discipline entry[26].

In conclusion, a model of Schumpeterian growth which incorporates some realistic features of the market for innovation like decreasing marginal productivity of investment, fixed costs and a first mover advantage for the incumbent monopolist, delivers realistic implications for the patterns of innovation. Monopolists do invest in R&D, even more than any other single firm, and their leadership persists with a certain probability, but sooner or later they are replaced by an outsider firm. This environment enhances growth and the aggregate probability of innovation.

The large investments in R&D by American firms, both leading corporations and smaller start ups in each sector, are probably at the source of much of the economic success of US and of its persistence. But US is also the leading economy in the world, whose innovations are spread around the world and whose market leaders are often global leaders. This increases further the value of innovations explaining the larger investment in R&D by American firms and especially by American main corporations.

[26] This is in contrast with the message of Segerstrom (2007), whose model of R&D investment by monopolists due to an exogenous technological advantage delivers the optimality of a negative R&D subsidy, even if the intuition for this outcome is unclear. This may show the importance of endogenizing monopoly persistence in a proper way.

VI - EXTENSIONS

The framework analyzed in this paper can be used for a number of macroeconomic investigations, and in this section I will try to provide the flavour of some of them (for a more extensive treatment see Etro (2006d).

I first focus on the same relation between growth and innovation. While a close attention has been paid to the effects of investments in innovation on growth, little interest has been captured by the opposite direction of causality: economic growth may affect the incentives to invest in R&D. This appears quite important for the empirical analysis on the determinants of growth (a serious critique to Schumpeterian growth models relies on the absence of a clear relation between R&D and growth) and also for growth accounting purposes[27]. I will address this issue augmenting the previous models with an external source of growth, which may just be the traditional exogenous technological progress or it may be endogenized through accumulation of human capital, public capital or other externalities increasing TFP. Then, I will evaluate the impact of this other source of growth on the incentives to invest and finally derive the nature of the relationship between overall growth and innovation. A surprising result emerges: even when innovation is the main engine of growth (in the sense that it actually contributes to most of the growth rate), growth and investment in innovation may be negatively correlated.

Assume that our parameter A grows at an exogenous rate $x = \dot{A}/A$: even if we refer to this as TFP, we may think of any source of growth which does not derive from endogenous innovation, for instance growth of foreign countries which spills over to the country. Now the expected discounted value of the profits with innovation k_j at time τ becomes:

$$V_\tau(k_j) = \frac{\mu\left(\dfrac{\alpha q^{k_j\alpha} A_\tau}{1+\mu}\right)^{\frac{1}{1-\alpha}} L}{r + p(k_j) - \dfrac{x}{1-\alpha}} \tag{31}$$

which is clearly increasing in x. Under this extension, the growth of output, as a function of the new source of growth, can be derived as $g(x) = \dot{Q}/Q + x/(1-\alpha)$, where the rate of technological progress is still defined by (14) but depends on x as well. As before, one can easily derive the equilibrium organization of the market for innovation under Nash or Stackelberg competition: now investment by each firm increases over time at a constant rate, while the number of firms is fixed and endogenously depends on the new source of growth. On the balanced growth path output must still grow at the same rate as consumption, which is given by the Euler equation (2). Hence, the equilibrium growth rate can be derived as:

$$g(x) = g(0) + \frac{x(1-\alpha+\alpha\ln q)}{(1-\alpha)(1-\alpha+\gamma\alpha\ln q)} \tag{32}$$

[27] See the pathbreaking work by Jones (1995) and the survey by Aghion - Howitt (1998, Ch.12) on empirical tests of the Schumpeterian growth theory and Barro - Sala-i-Martin (2004, Ch. 10) on its implications for growth accounting.

where $g(0)$ is the growth rate in absence of other sources of growth and the right hand side is increasing in x. Finally, we can residually derive the endogenous rate of technological progress as:

$$\frac{\dot{Q}}{Q} = g(0) + \frac{(1-\gamma)x\alpha \ln q}{(1-\alpha)(1-\alpha+\gamma\alpha \ln q)} \tag{33}$$

Hence, when $\gamma < 1$, an increase in growth due to other factors than innovation increases R&D investment: this happens through an increase in the number of firms investing in research (while the investment per firm does not change). However, when $\gamma > 1$, the increase in growth reduces entry and total investment in innovation. This surprising result is due to the effect of the endogenous adjustment of the interest rate on the value of innovations (31). When output growth increases, the interest rate must go up to raise savings and allow consumption growth to increase as well. The increase in the interest rate has a negative affect on the value of innovations and hence on entry of firms investing in R&D, and this negative effect must be compared with the positive direct effect exerted by the increase in x on the value of innovation. When the intertemporal elasticity of substitution is high (γ is small), a small increase in the interest rate is needed to clear the credit market and R&D investment unambiguously increases, but when the intertemporal elasticity of substitution is low (γ is large), the opposite happens and an increase in x reduces investment and technological progress. Summarizing, we have:

Proposition 4: An increase in the growth rate due to other factors than innovation reduces (increases) the number of firms investing in innovation and total investment in innovation when $\gamma > (<) 1$.

When the paradoxical negative relation between other factors of growth and innovation occurs, the empirical implications are quite dramatic: even if investment in innovation is the main engine of growth, the correlation between growth and investment/GDP ratio should be negative. Since realistic values for the intertemporal elasticity of substitution imply γ close to unity, we should not be surprised to find out that there is not a strong correlation between growth and R&D investment. In general, future empirical research on the relation between R&D and growth should take seriously the feedback effect of growth on R&D activity: the absence of a positive correlation between growth and R&D is not by itself a defeat of the Schumpeterian hypothesis. Etro (2006d) has extended the model to an open economy, providing implications for R&D policy coordination[28].

Finally, another interesting extension concerns the relation between endogenous growth and inflation. Endogenous growth models are well equipped to face monetary issues in growing economies, but they have been rarely used for this purpose. Here,

[28] See also Impullitti (2006a, b) on this. On related models of strategic export promotion see Kovac - Zigic (2006) and Boone - Ianescu - Zigic (2006).

building on an important contribution by Barro and Tenreyro (2006), I show that an inverted-U relation between inflation and long run growth emerges when monetary frictions affect the engine of growth.

Consider a closed economy, and introduce real money balances M/P, where M is nominal money issued at the growth rate $\sigma = \dot{M}/M$ and P is the price level for the final good, which is perfectly flexible and evolves with the rate of inflation $\pi = \dot{P}/P$. Adopting the Sidrausky approach with money in the utility function $C^{1-\gamma}/(1-\gamma) + \chi(M/P)^{1-\xi}/(1-\xi)$ with $\xi, \chi > 0$, standard optimization shows that the optimal consumption growth is still given by (2), while money demand is $M = P[\chi C^{\gamma}/(r+\pi)]^{1/\xi}$. Equating the latter and the money supply delivers the equilibrium price level at each point in time. This implies that on a balanced growth path the endogenous level of inflation is constant if and only if:

$$\pi = \sigma - \frac{\gamma g}{\xi} \tag{34}$$

In general equilibrium, monetary policy and hence inflation may affect the return rate and the growth rate. This is not the case when the prices of the intermediate goods perfectly adjust to changes in the price level of final goods: then inflation is superneutral. In such a case, the optimal monetary policy is always given by a generalized Friedman Rule which sets the nominal interest rate as close as possible to zero implying the optimal rate of money growth $\sigma = g\gamma(1-\xi)/\xi - \rho$ (which is positive for γ small enough).

However, when prices of intermediate goods are sticky in the newkeynesian tradition, new consequences emerge for equilibrium growth and we will focus on them. Before doing it, however, notice that, as emphasized by Barro and Tenreyro (2006), a surprise inflation temporary increases demand of intermediate goods and hence production. The effect of inflation in such a model suggests problems of time-inconsistency for a policy of price stabilization, as in the Barro-Gordon framework. However, our model generates a good reason for commitment to rigorous monetary rules: the reason is that inflation creates other effects on investment and these effects have permanent consequences on long run growth. There are many ways to formalize price stickiness. Following the newkeynesian literature, it is convenient to assume staggered pricesetting *`a la* Calvo, where the opportunity to adjust prices follows a stochastic process for each firm. However, our model provides an easy and realistic way to endogenize price adjustments assuming that they coincide with the introduction of new goods in the market. In particular, an innovator at time τ sets the price of its good at a level $P_\tau(1+\mu)$ and keeps it at this level until a new vintage is on the market with a new price. Then, the expected discounted value of the profits with an intermediate good k_j at time τ can be derived as:

$$V_\tau(k_j) = \int_\tau^\infty \left(\frac{\alpha A q^{k_j\alpha} P_t}{(1+\mu)P_\tau}\right)^{\frac{1}{1-\alpha}} \frac{L[(1+\mu)P_\tau - P_t]e^{-[r+p(k_j+1)](t-\tau)}}{P_t} dt \tag{35}$$

where $P_t = P_\tau e^{\pi(t-\tau)}$. Developing the integral, it can be verified that this value is an inverted-U curve in the inflation rate and it is maximized at some level $\hat{\pi}$, which is positive if μ is large enough. Competition to innovate is driven by (35) as before (even if firms now decide their real investment flows and change the nominal ones over time with inflation). Since the equilibrium effective discount rate, the aggregate investment and the growth rate of output are directly related with this value of the intermediate goods (35) by the free entry condition in the market for each innovation, they inherit the same non-monotonic relation with inflation. Hence, price stickiness, generates an inverted-U curve linking the inflation rate and the endogenous growth rate[29]:

$$g = g(\pi) \quad \text{with} \quad g'(\pi) \gtrless 0 \text{ for } \pi \lessgtr \hat{\pi} \tag{36}$$

The balance growth path is characterized by (34) and (36) for a given policy of constant money growth σ[30]. Summarizing, we have:

Proposition 5: In the presence of price stickiness, there is a long run inverted-U relation between inflation and growth due to the effects of expected inflation on the incentives to invest.

Notice that this result is in contrast with the Mundell-Tobin effect for which inflation stimulates investment and growth: here, this happens only for low levels of inflation, while high inflation erodes expected monopolistic profits and hence it reduces investment and growth. The last outcome may provide a channel for the negative relation between inflation and growth emphasized in the empirical literature at least for high levels of inflation.

While the newkeynesian theory has mainly focused on the short term consequences of inflation in presence of price frictions, little attention has been paid to the long run consequences, which can be even more important from a policy point of view. In our context, the utility maximizing policy can be quite complex, but if proper policies can solve the inefficiencies in the allocation of investment, the optimal inflation rate must be between the Friedman rule level and the growth maximizing level. Clearly, when $\chi \to 0$, so that the welfare costs from not implementing the Friedman rule under full price flexibility are negligible, the optimal policy tends to the growth maximizing one. Interestingly, the latter boils down to a policy of zero infla-

[29] This effect may be quite relevant. Imagine that prices are constant for a year, mark up is at 20% and inflation at 5%. Then, assuming $\alpha = 0.5$, after one year the flow of profits is reduced by more than 20%. If the average life-lenght of an intermediate good is one year (or prices are changed every year), inflation reduces the value of innovation by about 10%. As in the newkeynesian literature on businness cycles, small price frictions imply that demand shocks can have large macroeconomic consequences: however, here the consequences are permanent.

[30] For intermediate levels of money growth there are two equilibrium growth paths and the inefficient one is characterized by high inflation and low growth. This self-fulfilling stagflation has a simple intuition: if high inflation is expected, firms reduce investment decreasing output growth, which generates a low growth rate of consumption and money demand, which creates high inflation. However, such a path is unstable, and in what follows I focus on the stable path.

tion when pricing strategies are optimally chosen by monopolists[31]. If it is optimal to maximize the value of innovation and growth, the associated optimal rate of monetary growth is $\sigma^* = \gamma g(0)/\xi > 0$. Hence in this case we can conclude that when $\chi \to 0$ the optimal policy satisfies the Friedman rule under perfectly flexible prices but it requires price stabilization under sticky prices. For instance, an explicit solution for the long run growth rate as a function of the inflation rate can be obtained when $\alpha = 1/2$ and $\epsilon \to 1$:

$$g(\pi) = \frac{\pi + 1/2 + \sqrt{1/4 - \pi} - \rho}{\gamma + 1/\ln q} \tag{37}$$

which holds for small enough inflation and is clearly maximized by a policy of price stabilization.

The bottom line of this discussion is that whenever policy affects the value of innovations even marginally, it affects long run growth and hence it has permanent consequences. Here I focused on monetary policy, but fiscal policy affecting the monopolistic mark-ups or the return rate can have similar role as well.

VII - CONCLUSIONS

In this article I have developed a model of creative destruction where the engine of growth is in the microeconomic structure of the market for innovations as developed in the companion article Etro (2004). A main new result is about growth driven by market leaders which endogenously invest in R&D and persist in their leadership when facing free entry in the race for innovations: in such a case, growth driven by leaders is higher than growth driven by outsiders. This suggest that leading firms in high-tech sectors, like Intel for chips, Microsoft for software, IBM, Sony and Nokia for different kinds of hardware, and many others, have given and will give a fundamental contribution to technological progress and global growth, a point already made by Segerstrom (2007) in a related article about what he calls the Intel economics.On one side, this implies that the protection of intellectual property rights for

[31] To see this, notice that the optimal monopolistic mark up in sector j is:

$$\mu = \frac{(1-\alpha)\left[r + p(k_j+1)\right]}{\alpha\left[r + p(k_j+1) - \pi/(1-\alpha)\right]}$$

which is increasing in the inflation rate. The value of innovation then becomes:

$$V_\tau(\kappa_j) = \left(\frac{1-\alpha}{\alpha}\right)\left(\alpha^2 A q^{\kappa_j \alpha}\right)^{\frac{1}{1-\alpha}} L P_\tau \frac{\left[r + p(k_j+1) - \dfrac{\pi}{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}}}{\left[r + p(k_j+1) - \dfrac{\alpha\pi}{1-\alpha}\right]^{\frac{1}{1-\alpha}}}$$

which is maximized by $\hat{\pi} = 0$.

high-tech sectors is crucial to promote adeguately the incentives to invest in R&D to conquer the leadership and generate what I have called growth driven by market leaders. On the other side, antitrust authorites should be more careful in associating persistent leaderships in a high-tech sectors with dominant positions. In these sectros where competition is mostly *for* the market (rather than *in* the market), it is natural that better products conquer large shares and, exactly when entry is free, incumbent patent-holders have more incentives to invest and their leadership is more likely to persist. Hence there is no basis to relate in a significant way market shares and market power in dynamic sector. As I have emphasized elsewhere in a more general discussion (see Etro, 2007), industrial policy should primarily promote, and possibly subsidize, investment in R&D, while it should be less relevant whether the incumbent leaders or new comers invest in R&D and innovate once the competition for the market is open to entry.

Appendix: Growth with Full Persistence of Technological Leadership

In this appendix I provide further details on the model with Stackelberg leadership for monopolists in the R&D sector. The equilibrium derived in the text allows to derive the number of firms $n(\epsilon)$ as a function of ϵ and the equilibrium investments:

$$z(\kappa_j) = \frac{\epsilon\eta(\psi-\eta)}{[\psi-\epsilon(\psi-\eta)]}\left(\frac{\alpha A}{1+\mu}\right)^{\frac{1}{1-\alpha}} Lq^{\frac{\alpha}{1-\alpha}(\kappa_j+1)},$$

$$z_M(\kappa_j) = \frac{\epsilon\left[\psi-\mu+\eta q^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{1-\epsilon}}}{(1-\epsilon)q^{\frac{\alpha}{1-\alpha}}}\left(\frac{\alpha A}{1+\mu}\right)^{\frac{1}{1-\alpha}} Lq^{\frac{\alpha}{1-\alpha}(\kappa_j+1)}$$

The first is still smaller than the first best level, while the second one not necessarily. Moreover, the investment of the monopolist is increasing with ϵ and actually converging to ∞ for $\epsilon \to 1$. This implies that there is a cut-off $\hat{\epsilon}$ such that $n(\hat{\epsilon}) = 1$. Then for $\epsilon \geq \hat{\epsilon}$, the optimal strategy for the leader is pure entry deterrence (this applies a more general result in Etro, 2002). To derive the equilibrium under this regime of complete persistence of monopoly, notice that the investment of the leader must be slightly above the level at which the free entry condition allows entry by just one follower. Such an investment allows the leader to be alone in the patent race and, using the usual guess for the value function, it implies:

$$r + p = \epsilon^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}(\psi-\eta)$$

which using the equilibrium expression for the growth rate of consumption and income provides:

$$\psi = \eta + \frac{p\left[1 + \frac{\gamma\alpha \ln q}{1-\alpha}\right] + \rho}{\epsilon^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}}$$

This is a standard positive relation between the probability of innovation and the return from leadership. Since the leader is alone in the patent race, the probability of innovation is simply $p = \left[\phi(k_j)z_M(k_j)\right]^{\epsilon}$ which implies $z_M(k_j) = p^{(1/\epsilon)}/\phi(k_j)$. Now the Bellman equation expressing the value of leadership becomes:

$$\mathbf{V}(\kappa_j) = \frac{(\phi z_M)^{\epsilon}\mathbf{V}(\kappa_j + 1) + \pi(\kappa_j) - z_M}{r^s + p^s} - F(\kappa_j) =$$
$$= \frac{X(\kappa_j)}{r+p}\left[q^{\frac{\alpha}{1-\alpha}}p\frac{\psi}{r+p} + \mu - q^{\frac{\alpha}{1-\alpha}}p^{\frac{1}{\epsilon}} - \eta q^{\frac{\alpha}{1-\alpha}}\right]$$

which, using the guess value for $V(\kappa_j)$, provides:

$$\psi = \left[\frac{\mu - q^{\frac{\alpha}{1-\alpha}}p^{s\frac{1}{\epsilon}} - \eta q^{\frac{\alpha}{1-\alpha}}}{\rho - \frac{(1-\gamma)p\alpha \ln q}{1-\alpha}}\right]\left\{\rho + p\left[1 + \gamma\frac{\alpha \ln q}{1-\alpha}\right]\right\}$$

whose denominator must be positive under the transversality condition $\rho > (1-\gamma)g$, which requires γ large enough. This implies a negative relationship between the probability of innovation and the return from leadership due to the entry deterrence constraint: the larger is the investment in R&D needed to deter entry, the smaller is the value of being a leader. The two conditions above define the equilibrium values for ψ and p and hence for the interest rate r, the effective discount rate and the growth rate:

$$g = \frac{\epsilon^{\epsilon}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}(\psi - \eta) - \rho}{\gamma + (1-\alpha)/\alpha \ln q}$$

which are all decreasing in η and increasing in μ. It can be easily verified that approaching constant marginal productivity, the traditional case considered in the literature, that is for $\epsilon \to 1$ and $\eta \to 0$, the return from leadership ψ tends to μ and:

$$g\big\lfloor_{\epsilon\to 1, \eta\to 0} = \frac{\mu - \rho}{\gamma + (1-\alpha)/\alpha \ln q}$$

which is the same as under Nash competition in the market for innovations, a point already noticed by Robert Barro (2000, personal communication) and Cozzi (2007). When marginal productivity is constant in the R&D sector, incumbent monopolists with a first mover advantage in the patent races perfectly crowd out outsiders' investment resulting in no changes in the aggregate variables. A monopolist can deter entry in a patent race with constant returns to scale, but it will have to exhaust all the gains from a possible persistent leadership. This is not necessary under decreasing returns, that is way the return from leadership increases.

Under decreasing marginal productivity of R&D investment, the leader is investing just enough to deter entry, while it could marginally reduce its investment and allow entry by one follower, which would increase the aggregate probability of innovation, the effective discount rate, and hence also the growth rate. This implies that in the regime of entry deterrence, the equilibrium growth rate could be well below the one emerging without leaderships. As usual, it will be suboptimal if γ is small enough.

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