Global Innovation and R&D Policy Coordination

Federico Etro[1]

JEL Classification: O3, O4, F4
Key Words: Growth, Innovation

Abstract

I develop an open economy Schumpeterian growth model where fully fledged patent races drive investments in innovation and, when the incumbent patentholders are leaders in the patent races, they invest in R&D and their leadership persists and enhances growth. The model provides new insights on the growth process and, contrary to standard endogenous growth models, unambiguous policy conclusions about R&D policy. The equilibrium is characterized by a new form of dynamic inefficiency due to an inefficient bias toward too small firms in the market for innovation, and R&D subsidies are always part of the optimal innovation policy exactly because they help increasing the size of firms. However, even if the optimal unilateral strategic policy for each country requires positive subsidies, decentralized R&D policies do not enhance growth. In presence of trade frictions, the largest country has a comparative advantage in the innovation sector and, in the long run, it leads alone the technological frontier, exports intermediate goods, imports final goods and attracts foreign capital to finance investment. Also in such a more general framework, I emphasize a strong case for international R&D policy coordination.

[1]Catholic University of Milan (Italy), ECG and INTERTIC. I benefited from insightful discussions with D. Acemoglu, P. Aghion, R. Barro, M. Boldrin, D. de Meza, J. Harrington, D. Laibson, G. Mankiw, J. Zeira and K. Zigic. I am thankful to participants in seminars at Harvard, MIT, CERGE (Prague) and University of Milan. *E-mail*: fetro@intertic.org.

1 Introduction

In the last decade the theory of Schumpeterian growth managed to formalize in great depth the relation between investments in R&D and growth in closed and open economies+ and also to provide an accurate description of the endogenous incentives to invest in innovation and of the relationship between competition *in* the markets and *for* the markets on one side and growth on the other.[2]

Nevertheless, Schumpeterian theories were not successfull in providing clear suggestions on the policy side. In the endogenous growth models, the aggregate investment in R&D can be too large or too small,[3] and accordingly imply the need for R&D taxation or R&D subsidization. These ambiguous results are in strong contrast with the common view that investments in R&D are below their desirable level even in countries where they are particularly high. Moreover, in an international context investments in R&D generate leadership and comparative advantage in high-tech sectors, factors that are fundamental for long run growth.

For many reasons, private incentives to invest in risky R&D activities are limited in the real world and countries try to provide public support or subsidization to R&D and even to coordinate it within international unions. For instance, one of the main objectives of the European agenda is to increase R&D investment through subsidization and to coordinate such a policy across countries (see Katsoulacos, Tsipouri and Guy, 2005).

This paper offers a comprehensive motivation for international R&D coordination within a Schumpeterian growth model with multiple trading countries. Following recent advances in the empirical[4] and theoretical[5] literature, I introduce realistic features of the market for innovations as decreasing marginal productivity at the firm level and the possibility of wasteful duplications of resources between firms due to congestion reasons at the industry level, and take in consideration the possibility that incumbent patentholders have a competitive advantage in the patent races and hence endogenously invest in R&D.[6]

The model provides new insights on the growth process. The equilibrium is characterized by a new form of dynamic inefficiency due to a bias toward too small firms in the market for innovation. This implies that, in contrast with the ambiguous results of the standard endogenous growth literature, R&D subsidies are always part of the optimal innovation policy exactly because they help increasing the size of firms. Moreover, I derive the optimal unilateral R&D subsidies for a country whose firms engage in the international competition for innovating. Unfortunately decentralized R&D policies do not affect growth

[2]See Aghion and Howitt (2005) on the first and Etro (2004) on the latter.

[3]See Barro and Sala-i-Martin (2004).

[4]See Kortum [1993], Griliches [1994], Cohen and Klepper [1996].

[5]See Etro [2004] and Segerstrom [2006].

[6]For related and updated research on this topic see the International Think-tank on Innovation and Competition (www.intertic.org). For a recent survey of related theoretical and empirical advances on competition and growth see Aghion and Griffith (2005).

while they just shift profit from a country to another. This inefficiency creates a strong case for international coordination of R&D subsidization.

Finally, I study the effect of trade frictions in international trade, showing that in this stiylized model the largest country has a comparative advantage in the innovation sector and, in the long run, it tends to lead alone the technological frontier, export intermediate goods, import final goods and attracts foreign capital to finance investment. Also in such a more general framework, the strong case for R&D policy coordination is confirmed.

The paper is organized as follows. Section 2 introduces the general model. Section 3 solves it assuming that there are no trade frictions and derives the optimal R&D policy international coordination both in the cases when the market for innovations is characterized by Nash competition and by a more realistic Stackelberg competition with a leadership by the incumbent patentholders. Section 4 derives the optimal strategic trade policy for a country. Section 5 extends the model to effective trade frictions. Section 6 concludes.

2 The Model

Following the tradition started by Grossman and Helpman (1991), we will consider a general multicountry world with trade of inputs and final goods, and with innovations inducing growth. Each country $i = 1, 2, ..., M$ is populated by a representative agent with isoelastic utility of the standard kind:

$$U_i = \int_0^\infty \frac{C_i^{1-\theta}}{1-\theta} e^{-\rho t} dt \quad \text{with} \quad \theta > 0 \tag{1}$$

where $\rho > 0$ is the time preference rate.

Labour fource in country i is L_i. Labour does not move across countries while capital and final goods freely move across borders. Output Y_i can be used for consumption C_i, production of intermediate goods X_i or investment in R&D activities at the international rate of return r. In each country output is produced according to the constant return to scale function introduced by Barro and Sala-i-Martin [1995, Ch.7]:

$$Y_i = A_i L_i^{1-\alpha} \sum_{j=1}^{N} [q^{\kappa_j} X(k_j)]^\alpha \tag{2}$$

where A_i is Total Factor Productivity in country i, $X_i(\kappa_j)$ is the intermediate good j of quality κ_j used by country i, N is the constant number of intermediate goods, $q > 1$ parametrizes the increase in productivity of a new intermediate good and $\alpha \in (0, 1)$. The market for the final good, which is the *numeraire*, and the markets for labour and credit (to firms investing in R&D) are perfectly competitive.

In each country, the representative agent earns labour income and chooses consumption and savings according to the usual optimality condition:

$$\frac{\dot{C}_i}{C_i} = \frac{r-\rho}{\theta} \tag{3}$$

which holds at each point in time. Using the intertemporal resource constraint, one can derive an expression for savings depending on the expected value of national income and on the expected path of the interest rate.

Each intermediate good is produced at a unitary marginal cost and sold monopolistically until a newer version is on the market, which is a reasonable situation when the rate of creation of new products is fast enough. There can be frictions in trade of intermediates: imagine that for 1 unit of intermediate good sent to a foreign country, $d \leq 1$ units arrive at destination because of trade barriers associated with protectionism (but one may also think of losses due to incomplete protection of foreign IPRs) or just because of technological constraints in forms of iceberg transport costs . The parameter d can be interpreted as a measure of the degree of openness: when $d = 0$ the model boils down to a closed economy context, while for $d > 0$ we have trade between countries in inputs and final goods.

I will define $1 + \mu$ as the price for each monopolist (so that μ is the mark up), which may be microfounded as the monopolistic price $1/\alpha$ for drastic innovations, the limit price q for non drastic ones or in other ways (even taking into account other distortions, like taxation): in both cases, the price is the same for inputs sold at home or abroad. Demand in country i for the intermediate good j produced at home is:

$$X_i(\kappa_j) = \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu}\right)^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} L_i$$

while the demand from foreign country f is $X_f(\kappa_j) = d^{\frac{1}{1-\alpha}} X_j(\kappa_j)$. Summing over all countries, we have the total demand of intermediate good j,

$$X(\kappa_j) = \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu}\right)^{\frac{1}{1-\alpha}} \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f=1 f \neq i}^{M} A_f^{\frac{1}{1-\alpha}} L_f \right] \tag{4}$$

Defining $Q \equiv \sum_{j=1}^{N} q^{\frac{\kappa_j \alpha}{1-\alpha}}$ as an aggregate quality index and substituting in (2), we obtain the output of final goods of country i as:

$$Y_i = \left(\frac{\alpha}{1+\mu}\right)^{\frac{\alpha}{1-\alpha}} Q \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f=1 f \neq i}^{M} A_f^{\frac{1}{1-\alpha}} L_f \right] \tag{5}$$

Firms around the world invest to create new intermediate goods of higher quality. Investment in each sector depends on the expected value of innovations and on the interest rate profile, and it generates a probability of innovation $p(\kappa_j)$ in each sector, which implies an expected world technological progress $p\alpha \ln q/(1-\alpha)$, where $p \equiv \left[\sum_{j=1}^{N} p(\kappa_j) q^{\frac{\kappa_j \alpha}{1-\alpha}}\right]/Q$ is a weighted average of the probability of innovations. By (5), the average growth rate of income in each country output must be equal to this rate of technological progress:

$$\frac{\dot{Y}_i}{Y_i} = \frac{p\alpha \ln q}{1-\alpha} \tag{6}$$

A world market clearing condition equating investment and savings by all countries provides the international interest rate at each point in time and, given this, consumption and output independently follow (3) and (6) for each country. This would allow to characterize the behavior of the each economy in general, but here we will focus on the characterization of a balanced growth path where the international interest rate is constant.

To describe the investment side of the economy in such a balanced growth path, we need to describe the technology to create innovations. When an innovation for an intermediate good j generates the new quality rung κ_j, the innovator starts producing with the cutting-edge technology and obtains a flow of profits μ times the world demand of its input, that is $\mu X(\kappa_j)$. At the same time, the race to find out the subsequent innovation begins. To participate, a firm of country i has to pay a fixed cost $F(\kappa_j)$, which may include an entry fee set by the government, and spend a flow of resources $z_i(\kappa_j)$, which is the strategic choice of the firm.

Following the literature on patent races - for instance Etro [2004], the technology for the invention of new goods allows for decreasing marginal productivity at the firm level.[7] In particular, the investment for firm i gives birth to the innovation κ_j according to a Poisson process with arrival rate $p_i(\kappa_j)$ given by a concave function of $z_i(\kappa_j)$. To obtain closed form solutions, I assume that $p_i(\kappa_j) = [\phi(\kappa_j) z_i(\kappa_j)]^{\epsilon}$, where the function $\phi(\kappa_j)$ expresses how difficult is to discover technology κ_j and $\epsilon \in (0, 1]$ represents the degree of returns to scale in the innovation sectors or the elasticity of expected revenue with respect to the flow of investment. This parameter is unitary in the existent versions of the quality-ladder model, implying constant marginal productivity - equivalent to constant returns to scale since there is just one input - in the R&D sector, but empirical research, for instance by Cohen and Klepper [1996] and Kortum [1993] suggests an elasticity much smaller than 1.[8]

To simplify the presentation (and the notation), imagine that in the international competition for every innovation each firm is from a different country

[7]See also Aghion and Griffith (2005). A related investigation is present in a work by Zeira [2003]. However, his interest is in the choice of innovators between simple innovations and more difficult but radical innovations and across multiple research strategies.

[8]Segerstrom [2004] assumes that decreasing returns hold just for the incumbent monopolist, while constant returns to scale characterize all the other firms. He solves the model through simulations and assumes $\epsilon = 0.3$ as the average between the values proposed by Kortum.

(nothing would change if some country had more than one firm active in some sector). The arrival rate of innovation κ_j, is the sum of the individual arrival rates of the $n(\kappa_j)$ entrants plus the one of the incumbent, indexed with M,

$$p(\kappa_j) = \sum_{i=1}^{n(\kappa_j)} p_i(\kappa_j) + p_M(\kappa_j)$$

Using the properties of Poisson processes in a standard fashion, this implies that the expected discounted value of the profits of a firm from country i with innovation k_j is:

$$V_i(\kappa_j) = \int_0^\infty \mu \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu}\right)^{\frac{1}{1-\alpha}} \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f \neq i} A_f^{\frac{1}{1-\alpha}} L_f\right] e^{-[r+p(\kappa_j)]t} dt$$

$$= \frac{\mu \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu}\right)^{\frac{1}{1-\alpha}} \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f \neq i} A_f^{\frac{1}{1-\alpha}} L_f\right]}{r + p(\kappa_j)} \tag{7}$$

where $r + p(\kappa_j)$ is a sort of effective discount factor. Moreover, the expected net profit of entrant i in the patent race in sector j when the current quality is κ_j can be written as:

$$\Pi^i(k_j) = \frac{[\phi(\kappa_j) z_i(\kappa_j)]^\epsilon \mathbf{V}^M(\kappa_j + 1) - z_i(\kappa_j)}{r + p(\kappa_j)} - F(\kappa_j) \tag{8}$$

where $\mathbf{V}^M(\boldsymbol{\kappa}_j + 1)$ is the value of being monopolist with the next technology $\kappa_j + 1$. Since also the incumbent monopolist with the technology κ_j can invest to innovate, we need to consider its objective function, which is given by a recursive relation defining the value of being a monopolist:

$$\mathbf{V}^M(\kappa_j) = \max_{z_M \geq 0} \left\{ \frac{[\phi(\kappa_j) z_M]^\epsilon \mathbf{V}^M(\kappa_j + 1) - z_M}{r + p(\kappa_j)} + V(\kappa_j) - F(\kappa_j) \right\} \tag{9}$$

where the fixed cost is paid only if $z_M > 0$ and $V(\kappa_j)$ is given by (7). What drives investment and growth is exactly the attempt to conquer this value. In the standard literature, monopolists do not invest, hence the value of leadership is just the expected profits from the next innovation. However, as we will see later on, monopolists invest when they have a leadership in the patent race and in that case, the value of the innovation includes also the option value of a persistent monopoly, which fundamentally modifies the incentives to do research. Finally, I will assume that new ideas are more difficult to obtain when the scale of the sector increases, and that the fixed cost is a constant fraction of the expected cost of production with the new technology:

$$\phi(\kappa_j) = \left[(\alpha q^{\kappa_j \alpha})^{\frac{1}{1-\alpha}} \left(\sum_{f=1}^{M} A_f^{\frac{1}{1-\alpha}} L_f\right)\right]^{-1}$$

$$F(\kappa_j) = \eta \int_0^\infty X(\kappa_j + 1) e^{-[r+p(\kappa_j+1)]t} dt \qquad (10)$$

with $\eta \in (0, \mu)$. I want to capture the idea that the larger is the scale of expected production of a firm, the larger are the costs necessary to discover and develop the associated technology (construction of prototypes and samples, new assembly lines and training of workers). These assumptions will deliver a balanced growth path and will avoid scale effects on the equilibrium growth rate, in line with the last generation of quality-ladder models - see Jones [1995] and Barro and Sala-i-Martin [2004].

With this framework we can now solve for the equilibrium and for the optimal policies. For expository purposes it is convenient to focus first on the case of a world without trade barriers ($d = 1$) and leave for Section 5 the extension of the model to effective trade barriers.

3 World Growth and R&D Coordination

In this section I will derive the international decentralized equilibrium in absence of trade barriers and describe how countries can coordinate their policies to achieve the optimal allocation of resources. In the next section I will study what are the incentives of single countries to adopt R&D policies unilaterally.

Consider a decentralized international market for global innovations. As well known, under free entry the leader does not invest, because its best strategy is to stay out from the patent race and enjoy the profits from its current product until a new innovation will make it obsolete. Competition for innovations is just between outsiders and the scope of this section is to characterize the equilibrium organization of investment - the number of firms and the size of their investments - together with the usual macroeconomic variables and to derive the optimal coordination of R&D policies across countries.

In each sector, the lack of investment by incumbents implies that the value of being a monopolist (9) boils down to:

$$\mathbf{V}^M(\kappa_j) = V_i(\kappa_j) = \frac{\mu \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu} \right)^{\frac{1}{1-\alpha}} \left[\sum_f A_f^{\frac{1}{1-\alpha}} L_f \right]}{r + p\left(k_j\right)} \qquad (11)$$

for any i: in this ideal world, the value of an innovation is the same for any firm. This implies that firms from every country will invest in R&D and technological improvements will spread across all countries. As we saw, also growth is the same everywhere, since its only source is technological progress. However, income is different across countries and there is intra-industry trade between them - as in Krugman [1980]. In particular, if I is the set of intermediate goods produced by firms of country i, the value of its imports will be:

$$IMP_i = (1+\mu)\left(\frac{\alpha}{1+\mu}\right)^{\frac{1}{1-\alpha}}(Q-Q_i)S_i \quad \text{with } Q_i \equiv \sum_{j\in I} q^{\frac{\kappa_j\alpha}{1-\alpha}} \text{ and } S_i \equiv A_i^{\frac{1}{1-\alpha}} L_i$$

where Q_i is the quality index for the goods produced in country i and $Q \equiv \sum_{i=1}^{M} Q_i$. Imports are higher for countries engaging in less R&D activity and for larger economies, or more precisely with higher TFP and larger populations. The value of exports is:

$$EXP_i = (1+\mu)\left(\frac{\alpha}{1+\mu}\right)^{\frac{1}{1-\alpha}} Q_i(S-S_i)$$

which is lower for countries with higher productivity and size. Finally the trade balance becomes:

$$TB = \left[\frac{\alpha}{(1+\mu)^{\alpha}}\right]^{\frac{1}{1-\alpha}}(Q_iS - QS_i) \tag{12}$$

This implies that a country can run a trade surplus only if it engages in more R&D investments than average compared to the size of its economy. The empirical implication is that:

when the trade balance is even, the correlation between R&D investments and the size of economies should be positive.

At each point in time, the corresponding trade surplus in intermediate goods, the national savings and the returns on foreign assets will have to be matched by a deficit in trade of final goods, by investments in R&D and increases in the foreign net assets (that is current account surpluses). Since the international credit market works perfectly, investors are indifferent between investing in domestic firms or buying foreign ones and the international location of R&D activities is indeterminate. Richer countries will invest more, but they may well do it through foreign direct investment. Moreover, technological leadership shifts from a country to another one in each sector.

Let us return now to the equilibrium analysis for the market for innovations. Each firm chooses its investment $z_i(\kappa_j)$ to maximize (8) taking as given the strategies of the other firms, the value of the next innovation and the international interest rate, while the free entry condition sets the expected profits (8) equal to zero providing the equilibrium number of entrants $n(\kappa_j)$. Combining the optimality condition and the free entry condition we obtain the investment per firm:

$$z(\kappa_j) = \epsilon^{\frac{1}{1-\epsilon}}\phi(\kappa_j)^{\frac{\epsilon}{1-\epsilon}}\left[V(\kappa_j+1) - F(\kappa_j)\right]^{\frac{1}{1-\epsilon}} \tag{13}$$

which is increasing in the value of the innovation net of the fixed cost of entry, while it is independent from the interest rate.

Using the endogenous value of innovation (7) and our assumptions (10) in (13) and substituting in the free entry condition, which sets (8) equal to zero, we can express the probability of innovation as a linearly decreasing function of the interest rate:

$$\begin{aligned} p(k_j) &= \frac{[\phi(k_j)z(k_j)]^{\epsilon}\, V(k_j+1) - z(k_j)}{F(k_j)} - r = \qquad (14) \\ &= \frac{[\phi(k_j)X(k_j+1)]^{\frac{\epsilon}{1-\epsilon}}\left[\epsilon^{\frac{\epsilon}{1-\epsilon}}[\mu-\eta]^{\frac{\epsilon}{1-\epsilon}}\mu - \epsilon^{\frac{1}{1-\epsilon}}[\mu-\eta]^{\frac{1}{1-\epsilon}}\right]}{\eta\,[r+p(k_j)]^{\frac{\epsilon}{1-\epsilon}}} - r = \\ &= \left[\frac{\epsilon\,(\mu-\eta)}{(1+\mu)^{1/(1-\alpha)}}\right]^{\epsilon}\left[\frac{\mu-\epsilon\,(\mu-\eta)}{\eta}\right]^{1-\epsilon} - r \quad \text{for any } k_j \end{aligned}$$

Hence total investment in each patent race is decreasing in the interest rate as well. Using this to explicit the expected value of innovation (7), and substituting in (13) again, we obtain the equilibrium flow of investment per firm:

$$z(\kappa_j) = \frac{\epsilon\eta\,(\mu-\eta)}{[\mu-\epsilon\,(\mu-\eta)]}\left(\frac{\alpha q^{\alpha(\kappa_j+1)}}{1+\mu}\right)^{\frac{1}{1-\alpha}}\left(\sum_f A_f^{\frac{1}{1-\alpha}} L_f\right) \qquad (15)$$

which is increasing in the quality achieved in the single sector, since this implies higher demand and hence higher expected profits for the corresponding intermediate product, and increasing in the degree of returns to scale, ϵ, since this makes investment more productive. For a given scale of production, investment is also increasing in the mark up μ, which is exactly the core of the Schumpeterian idea that monopolistic profits drive investment of single firms.[9] Finally, the interest rate does not affect the individual investment of firms, while it negatively affects the number of international active firms and hence the total investment.

On a balanced growth path with a constant interest rate, the world resource constraint implies that income, investment and hence consumption must grow at the same rate. Equating (3) and (6) and using the aggregate probability of innovation common to all sectors and given by (14), we can derive all the equilibrium variables. In particular, the equilibrium growth rate is:

$$g = \frac{\left[\frac{\epsilon(\mu-\eta)}{(1+\mu)^{1/(1-\alpha)}}\right]^{\epsilon}\left[\frac{\mu(1-\epsilon)+\epsilon\eta}{\eta}\right]^{1-\epsilon} - \rho}{\theta + (1-\alpha)\,/\alpha\ln q} \qquad (16)$$

As one could expect, the more costly are innovations (higher η), the lower is equilibrium growth, while the relation between growth and ϵ is U-shaped.

[9] The effect of higher fixed costs on investment can be shown to be non monotonic, positive for η low but negative for η high enough: on one side high fixed costs reduce expected profits for a given life of the patent, but on the other, they reduce the innovation rate in the future so as to increase the expected life of the patent.

The equilibrium arrival rate of innovations is directly proportional to the above growth rate, while the equilibrium number of firms is:

$$n = \frac{\left[\frac{\mu-\epsilon(\mu-\eta)}{\eta}\right] - \rho\left[\frac{(1+\mu)^{1/(1-\alpha)}[\mu-\epsilon(\mu-\eta)]}{\epsilon\eta(\mu-\eta)}\right]^{\epsilon}}{1+\alpha\theta\ln q/(1-\alpha)} \tag{17}$$

hence higher size innovations are associated with higher growth but fewer firms (and less frequent innovations).[10]

3.1 The social planner solution

The decentralized equilibrium does not optimizes the internatioanl allocation of investment across firms and countries in the sense that it does not minimizes R&D expenditure for a given probability of innovation. Here we will derive the optimal allocation of resources which maximizes world welfare. Let us focus first on the optimal organization of the world market for innovations. It is immediate to derive from the concavity of the arrival rate that it is optimal to allocate equal flows of investment between all the R&D firms active around the world in a specific sector. Consider linear investment flows in the future scale of production, as:

$$z(\kappa_j) = \beta X(\kappa_j + 1) = \beta q^{\frac{\alpha}{1-\alpha}} X(\kappa_j)$$

for each firm in sector j, where the paremeter β must be chosen optimally. Let us keep the production of intermediates at the level chosen by the monopolist in the decentralized equilibrium.[11]

The resource constraint of the world must take into account the fixed costs, which are paid only at the beginning of each new patent race. Without loss of generality let us assume that the world devotes a flow of resources for this purpose in each sector. If the number of sectors N is high enough, one can approximate this flows, say $f_j(\kappa_j)$ with those equating their expected present value $f_j(\kappa_j)/[r+p(k_j)]$ to the fixed cost $F(\kappa_j)$, that is, using (10), with $f_j(\kappa_j) = \eta X(\kappa_j + 1)$. Using the expressions for the quantity of intermediate

[10] In equilibrium both n and g are suboptimal at least for θ small enough. This result has a simple intuition: when the intertemporal elasticity of substitution is large (θ is low), it is optimal to choose a high growth rate of consumption, hence the social value of innovations is high. On the other side, the private value of innovations depends on market features which are independent from consumers preferences (except for an indirect channel going through the interest rate). Hence, for low enough θ, the social value of innovations is larger enough than the private value and the optimal number of firms becomes larger than the equilibrium number.

[11] As well known, a social planner would not distort the choice of the input mix, hence we are basically solving for a second best allocation (the first best allocation would be obtained by subsidizing monopolists in such a way that their price equates marginal cost).

goods and for the output, we can rewrite the world resource constraint as:

$$\sum_{i=1}^{M} Y_i = \frac{X}{\alpha} = \sum_{i=1}^{M} C_i + \sum_{j=1}^{N} X_j(\kappa_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} z_i(\kappa_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} f_j(\kappa_j) = \tag{18}$$

$$= \sum_{i=1}^{M} C_i + X\left[1 + n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]$$

from which we derive an expression for world consumption holding at each point in time. Under the optimal allocation of resources, world growth is determined by the rate of innovation as:

$$g = n\left[\phi(\kappa_j)z(\kappa_j)\right]^{\epsilon}\frac{\alpha \ln q}{1-\alpha} \approx \left(\frac{\beta}{(1+\mu)^{1/(1-\alpha)}}\right)^{\epsilon}\frac{\alpha n \ln q}{1-\alpha} \tag{19}$$

Consumer preferences are identical across countries. Hence, given a constant growth rate of consumption, intertemporal utility for the representative world agent is finite as long as $\rho > (1-\theta)g$ and can be written as:

$$U = \int_0^{\infty}\frac{C_{it}^{1-\theta}}{1-\theta}e^{-\rho t}dt = \frac{C_0^{1-\theta}}{(1-\theta)\left[\rho-(1-\theta)g\right]} \tag{20}$$

Finally, using (18) and (19) in (20), we can summarize the social planner problem as:

$$\max_{n,\beta}\frac{\left[\frac{1-\alpha}{\alpha} - n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]^{1-\theta}}{(1-\theta)\left[\rho-(1-\theta)\left(\frac{\beta}{(1+\mu)^{1/(1-\alpha)}}\right)^{\epsilon}\frac{\alpha n}{1-\alpha}\ln q\right]} \tag{21}$$

which puts in clear evidence the basic trade-offs. A higher number of firms or a higher flow of investment per firm imply a higher growth rate of consumption but with a lower initial consumption level (and the time preference rate and the elasticity of substitution govern this trade-off in a standard fashion), but the weights on benefits and costs are different for the two choice variables.[12] If an interior solution exists, dividing the first order conditions for the social planner problem (21) with respect to β and n,[13] we can easily derive $\beta^* = \epsilon\eta/(1-\epsilon)$,

[12] The higher is the fixed cost parameter η the more costly is to increase the number of firms rather than the flow of investment. Finally, also the size of the innovations q and the parameter α characterizing the elasticity of demand of intermediate goods influence the trade-off.

[13] The first order conditions for the social planner problem (21) with respect to β and n are:

$$q^{\frac{\alpha}{1-\alpha}}\left[\rho-(1-\theta)\frac{\beta^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\frac{\alpha n\ln q}{1-\alpha}\right] = \epsilon\frac{\beta^{\epsilon-1}\alpha}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\frac{\ln q}{1-\alpha}\left[\frac{1-\alpha}{\alpha}-n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]$$

$$(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\left[\rho-(1-\theta)\frac{\beta^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\frac{\alpha n}{1-\alpha}\ln q\right] = \frac{\alpha\beta^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\frac{\ln q}{1-\alpha}\left[\frac{1-\alpha}{\alpha}-n(\beta+\eta)q^{\frac{\alpha}{1-\alpha}}\right]$$

which implies the optimal flow of investment in R&D per firm:

$$z^*(\kappa_j) = \frac{\epsilon\eta}{1-\epsilon}\left(\frac{\alpha}{1+\mu}\right)^{\frac{1}{1-\alpha}} q^{\frac{\alpha(\kappa_j+1)}{1-\alpha}} \left(\sum_f A_f^{\frac{1}{1-\alpha}} L_f\right) \tag{22}$$

Finally we obtain the optimal number of R&D laboratories as:

$$n^* = \frac{\left[\left(\frac{1-\epsilon}{\eta}\right)\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\left(\frac{1-\alpha}{\alpha}\right) - \rho\left(\frac{(1-\epsilon)(1+\mu)^{\frac{1}{1-\alpha}}}{\epsilon\eta}\right)^{\epsilon}\right]}{\theta\left[q^{\frac{\alpha}{1-\alpha}} - 1\right]} \tag{23}$$

which is decreasing in ϵ at least for ϵ high enough: this implies that when the marginal productivity of the investment is high enough, it is optimal to have just one laboratory investing in R&D. Finally, substituting β^* and n^* in our expression for growth (19), we obtain that the optimal world growth rate is:

$$g^* = \frac{1}{\theta}\left[\frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left[1-q^{-\frac{\alpha}{1-\alpha}}\right]\left(\frac{1-\alpha}{\alpha}\right) - \rho\right] \tag{24}$$

which is higher than the equilibrium growth rate for any θ smaller than a cut-off.

Finally, let us compare the decentralized equilibrium with the social planner solution. The optimal investment per firm (22) is always larger than (15): in a decentralized equilibrium firms tend to choose inefficiently small investments. The intuition relies on the fact that researchers do not internalize the effect of their choices on the entry decision, and entry creates wasteful duplication of R&D expenditures, in terms of fixed costs of research: hence, *ceteris paribus*, firms choose suboptimal investment. Since growth depends on the probability of innovation, the decentralized equilibrium is dynamically inefficient:

the world economy could achieve the same aggregate probability of innovation investing a smaller amount of total resources or increase the former at the same level of the latter.

Notice that this form of dynamic inefficiency is absent in traditional models of endogenous growth, where the economy may grow above or below an optimal benchmark, but cannot increase the growth rate without giving up to some of the current consumption: when marginal productivity in the R&D sector is decreasing, the endogenous organization of this sector creates this inefficiency.

3.2 International coordination of R&D policies

Only proper R&D policies can solve the dynamic inefficiency problem of the world economy. Imagine that countries could coordinate on a policy of R&D subsidization: for any unit of investment, there is a subsidy at rate s financed with lump sum taxes. Clearly such a policy would increase investment per firm.

Reworking the above derivation, one can obtain the equilibrium investment as a function of the subsidy,

$$z(\kappa_j, s) = \frac{z(\kappa_j, 0)}{1-s} > z(\kappa_j, 0) \tag{25}$$

which is an increasing and convex function of the subsidy rate. Using (22) and (25), one can easily derive the subsidy which induces the optimal investment per firm in the decentralized equilibrium:

$$s^* = \frac{\eta}{\mu(1-\epsilon) + \eta\epsilon} \in (0,1) \tag{26}$$

which is always positive, decreasing in μ and increasing in η, since higher effective markups already create larger investments.

The dynamic inefficiency of the growth process shows that a country with an industrial structure characterized by small firms achieves inefficient results, and could grow more without losses in current consumption if its firms were increasing in size. This general conclusion may shed new light on the problems of countries that do not grow much and lack large and innovative corporations. This is the case of many European countries, most notably of Italy, whose industrial structure is characterized by a large number of small and medium size enterprises whose innovative capacity is quite limited.[14]

The fully optimal international coordination can be achieved with two policy tools, a positive R&D subsidy, which optimally allocates resources between investors and an entry fee increasing the cost of entry, which targets the optimal number of firms.[15] To derive the optimal R&D policy let us introduce, together with the subsidy rate s, an entry fee which is the fraction τ of expected production costs in each patent race. The equilibrium growth rate becomes:

$$g(s,\tau) = \frac{\left[\frac{\epsilon(\mu-\eta-\tau)}{(1+\mu)^{\frac{1}{1-\alpha}}(1-s)}\right]^{\epsilon}\left[\frac{\mu(1-\epsilon)+\epsilon(\eta+\tau)}{\eta}\right]^{1-\epsilon} - \rho}{\theta + (1-\alpha)/\alpha \ln q} \tag{27}$$

The optimal R&D policy is given by (s^*, τ^*) such that $z(k)$ and $g(s,\tau)$ equate $z^*(k_j)$ and g^*, that is:

$$s^* = \frac{1}{\frac{\mu}{\eta+\tau^*}(1-\epsilon)+\epsilon} \in (0,1) \tag{28}$$

[14] For recent policy analysis on the benefits of market reforms on growth taking in considerations the market for innovations, see Faini *et al.* [2004]. At a theoretical level see Grieben [2005].

[15] Only in the limiting case of constant returns to scale, which is the traditional focus of this literature, the size and the number of firms do not matter and an R&D subsidy alone can achieve optimality. Notice that approaching constant returns to scale in our model (that is when $\epsilon \to 1$ and $\eta \to 0$), the investment by each firm and the number of firms become indeterminate, but the equilibrium growth rate converges to the traditional one (see Barro and Sala-i-Martin [2004]): $g \to \left[\mu/\bar{\zeta}(1-s) - \rho\right] / \left[\theta + (1-\alpha)/\alpha \ln q\right]$.

$$\tau^* = \frac{\left[\mu(1-\epsilon)+(\eta+\tau^*)\epsilon\right]^{\frac{1}{1-\epsilon}}\left[\frac{\epsilon}{(1+\mu)^{\frac{1}{1-\alpha}}(1-\epsilon)}\right]^{\frac{\epsilon}{1-\epsilon}}}{\left[1+(1-\alpha)/\theta\alpha\ln q\right]\left[\frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\left(\frac{1-\alpha}{\alpha}\right)-\rho\right]+\rho}-\eta \tag{29}$$

which provide two unique optimal policy tools. Clearly $g^* < g(s,0)$ for θ high enough, in which case, the optimal entry fee is positive. Summarizing,

international coordination of R&D policies requires a positive R&D subsidy to investment and an entry fee (positive for θ large enough) to achieve the optimal organization of the market for innovations and the optimal growth rate.

3.3 Innovation by leaders and R&D coordination

Many product innovations are due to dominant firms and a lot of the investment in R&D is actually done by both incumbent monopolists and new firms.[16] Existing models about innovation and growth are inconsistent with this simple fact, since under Nash competition and free entry, as we have seen also in the previous section, an incumbent monopolist has no incentives to invest in R&D. Recent research has rationalized investment of the incumbents in a partial equilibrium framework showing that monopolists invest in R&D more than any other firm as long as they are leaders in the sense of Stackelberg in markets for innovations where entry is free.[17] The requirement that incumbents are leaders in the market for innovations is realistic: after all, it is reasonable to imagine that they have a credible commitment to invest a certain amount of resources to improve their own products and protect their own rents. Otherwise, we can imagine that incumbent monopolists can undertake some preliminary investments which affect their profitability from engaging in R&D activity, like building laboratories, hiring researchers or borrowing to invest.[18]

Let us consider the market for innovation described in Section 2, where (8) and (9) are the objective functions of the entrants and the leader, and the latter has a first mover advantage. In this set up, analyzed in more detail in the Appendix, the partial equilibrium for each sector is characterized as follows: under Stackelberg competition in the market for innovations, when ϵ is large enough incumbent monopolists deter entry obtaining complete persistence of

[16] For empirical evidence on this classic Schumpeterian insight see for instance Blundell *et al.* [1999] and Segerstrom [2006].

[17] See Etro [2004]. This behaviour of the leaders under free entry is a particular case of a much more general result established in Etro [2002] where I have shown that Stackelberg leaders are always aggressive (under quantity or price competition or in patent races as here) whenever entry is endogenous. For related theoretical models see Zigic *et al.* [2005] and Wiethaus [2005].

[18] As I have shown in a more general context (see Etro [2006a]), this kind of strategic investment allows to reproduce similar outcomes to Stackelberg equilibria with free entry: leaders always overinvest strategically to be aggressive in the market for innovations afterward. The policy implications of this theories for industrial policy are discussed in Etro (2006b).

their leadership, while for smaller values of ϵ they allow entry but invest more than any outsider. In what follows, I will focus on the latter case, which is more realistic.[19]

When marginal productivity is decreasing (enough), the free entry condition pins down the number of followers in each sector. In this case, it is easy to verify that their optimal strategy $z(k_j)$ is always independent from the one of the leader (while the number of followers decreases in the investment of the leader), hence the effective discount rate $r+p(k_j)$ must be also independent from the leader's strategy [Etro, 2004]. The leader chooses its investment $z_M(k_j)$ to solve the problem (9), where the effective discount rate is independent from its choice and hence taken as given. From the first order conditions of the leader and of the followers and the zero profit condition, we obtain:

$$z(\kappa_j) = \frac{\left\{\epsilon\phi(\kappa_j)\left[\mathbf{V}^M(\kappa_j+1) - F(\kappa_j)\right]\right\}^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)} < z_M(\kappa_j) = \frac{\left[\epsilon\phi(\kappa_j)\mathbf{V}^M(\kappa_j+1)\right]^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)} \tag{30}$$

The characterization of the equilibrium is complicated from the fact that now we do not know what is the value of being a monopolist, since this is the solution to the recursive relation (9). Here I provide dynamic programming techniques to solve analytically this problem, which will emerge whenever one is dealing with Schumpeterian models of growth where incumbent monopolists engage in R&D activity - see also Segerstrom and Zelnieker [1999] and Segerstrom [2006]. To derive the balanced growth path and the equilibrium value function $\mathbf{V}^M(\kappa_j)$, the functions $z(k_j)$ and $z_M(k_j)$ and the equilibrium values for g, r, p and n, we can adopt the method of undetermined coefficients. Let us guess a functional form for the value function as:

$$\mathbf{V}^M(\kappa_j) = \mathbf{V}^M(\kappa_j - 1)q^{\frac{\alpha}{1-\alpha}} = \psi\frac{X(\kappa_j)}{r+p} \tag{31}$$

where ψ is a coefficient to be determined which can be interpreted as the rate of return from leadership. This must be larger than μ, otherwise the value of being a leader investing in R&D would be smaller than the value of being a leader without investing (i.e.: it would be optimal to stay out of the patent race for the leader). Substituting in (30), this implies:

$$z(\kappa_j) = \left(\frac{\epsilon(\psi-\eta)}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}(r+p)}\right)^{\frac{1}{1-\epsilon}} q^{\frac{\alpha}{1-\alpha}}X(\kappa_j), \quad z_M(\kappa_j) = z(\kappa_j)\left(\frac{\psi}{(\psi-\eta)}\right)^{\frac{1}{1-\epsilon}} \tag{32}$$

and, using this in the recursive relation for the value of innovating, we have:

$$\mathbf{V}^M(\kappa_j) \quad = \quad \frac{[\phi(\kappa_j)z_M(\kappa_j)]^{\epsilon}\mathbf{V}^M(\kappa_j+1) - z_M(\kappa_j)}{r+p} + V(\kappa_j) - F(\kappa_j) \tag{33}$$

[19] Segerstrom [2006] has criticized my approach for implying a low persistence of monopolies. However, my approach is even consistent with complete persistence (for ϵ high enough). Nevertheless, in a realistic set up monopolies should be persistent, not eternal.

$$= \frac{X(\kappa_j)}{r+p}\left[\mu + \left(\frac{\epsilon}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\right)^{\frac{\epsilon}{1-\epsilon}}\left(\frac{\psi}{r+p}\right)^{\frac{1}{1-\epsilon}} q^{\frac{\alpha}{1-\alpha}}(1-\epsilon) - \eta q^{\frac{\alpha}{1-\alpha}}\right]$$

whose right hand side contains the sum of the mark up from the current innovation and another term which represents the option value of remaining the monopolist after the next innovation: this option value is positive because of the leadership advantage. Using (31) and solving (33) for the effective discount rate we have:

$$r+p = \frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left[\frac{(1-\epsilon)\, q^{\frac{\alpha}{1-\alpha}}}{\psi - \mu + \eta q^{\frac{\alpha}{1-\alpha}}}\right]^{1-\epsilon}\psi \tag{34}$$

which provides a negative relation between the effective discount rate $r+p$ and the rate of return from leadership ψ: *the higher is the effective discount rate, the shorter is the lifetime of an innovation, and hence the lower is the value from being a leader.*

Moreover, the zero profit condition for the followers provides another expression for the effective discount rate which is analogous to (14):

$$r+p = \frac{\epsilon^{\epsilon}(\psi-\eta)^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left[\frac{\psi(1-\epsilon)+\epsilon\eta}{\eta}\right]^{1-\epsilon} \tag{35}$$

This is a positive relation between the effective discount rate $r+p$ and the rate of return from leadership ψ: *the higher is the value of being a leader, the larger will be the investment in R&D and hence the probability of innovation and the effective discount rate.*

Equating (34) and (35) we obtain the unique equilibrium value for ψ which provides all the equilibrium relations. An implicit expression for ψ is given by:

$$\psi = \mu + \frac{(1-\epsilon)\,\eta q^{\frac{\alpha}{1-\alpha}}\psi^{\frac{1}{1-\epsilon}}}{(\psi-\eta)^{\frac{\epsilon}{1-\epsilon}}\left[\psi(1-\epsilon)+\epsilon\eta\right]} - \eta q^{\frac{\alpha}{1-\alpha}} > \mu \tag{36}$$

which immediately implies that under Stackelberg competition in the market for innovations, the equilibrium rate of return from leadership is higher than under pure Nash competition because of the option value of monopoly persistence.

The equilibrium effective discount rate derived above allows to explicit the growth rate as:

$$g = \frac{r+p-\rho}{\theta + (1-\alpha)/\alpha \ln q} \tag{37}$$

and then to derive the number of firms $n(\epsilon)$ as a function of ϵ and the equilibrium investments:

$$z(\kappa_j) = \frac{\epsilon\eta(\psi-\eta)}{[\psi - \epsilon(\psi-\eta)]}\left(\frac{\alpha q^{\alpha(\kappa_j+1)}}{1+\mu}\right)^{\frac{1}{1-\alpha}}\left(\sum_f A_f^{\frac{1}{1-\alpha}} L_f\right),$$

$$z_M(\kappa_j) \quad = \quad \frac{\epsilon\left[\psi-\mu+\eta q^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{1-\epsilon}}}{(1-\epsilon)q^{\frac{\alpha}{1-\alpha}}}\left(\frac{\alpha q^{\alpha(\kappa_j+1)}}{1+\mu}\right)^{\frac{1}{1-\alpha}}\left(\sum_f A_f^{\frac{1}{1-\alpha}}L_f\right)$$

which are both smaller than the optimal level.

The investment of the leader is increasing with ϵ and actually converging to ∞ for $\epsilon \to 1$. This implies that there is a cut-off $\hat{\epsilon}$ such that $n(\hat{\epsilon}) = 1$. Then, for $\epsilon \geq \hat{\epsilon}$ the optimal strategy for the leader is pure entry deterrence - this applies a more general result by Etro [2002,a]. Using (35), this implies that the effective discount rate and hence both the growth rate and the aggregate probability of innovation must be higher than under Nash competition.[20] The incumbency advantage adds a "turbo" to the engine of world growth because it endogenously increases the value of innovations associating with them an option to persistent leadership, which increases aggregate investment and hence growth. Moreover, we can easily verify that both the return from international leadership ψ, the effective discount rate and hence the growth rate are increasing in the mark up μ. An increase in the fixed cost of innovation through η decreases the effective discount rate and hence the growth rate of the economy, but it has ambiguous effects on the value of being a leader. In conclusion, for ϵ small enough, under Stackelberg competition in the market for innovations, monopolists invest in R&D more than any outsider and the equilibrium growth rate is:

$$g = \frac{\frac{\epsilon^\epsilon}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}\left[\frac{\eta q^{\frac{\alpha}{1-\alpha}}}{\psi-\mu+\eta q^{\frac{\alpha}{1-\alpha}}}\right]^{1-\epsilon}\psi-\rho}{\theta+(1-\alpha)/\alpha\ln q} \tag{38}$$

where ψ, given by (36), is decreasing in ζ and η, and increasing in μ. Most of all, under Stackelberg competition in the market for innovations, the equilibrium growth rate is higher than under pure Nash competition:

innovation by world leaders increases the incentives to invest in R&D in each sector so as to enhance the world growth rate.

Clearly, when the engine of growth is given by persistent monopolistic positions as in this model, the investment by each firm increases, but it is still below the optimal level for both the incumbent monopolists and the outsiders: the dynamic inefficiency is still present. It can be shown that the optimal allocation of resources can be achieved with a positive subsidy for the entrants, a smaller but positive subsidy for the incumbent monopolists and an appropriate entry fee to discipline entry. Efficiency would require $z(\kappa_j) = z_M(\kappa_j)$ and hence $(\psi-\eta)/(1-s) = \psi/(1-s_M)$, where s is the subsidy for the followers and s_M the

[20] This does not need to be the case when incumbents find optimal to deter entry (that is for high enough ϵ). Paradoxically, even if the rate of return from leadership is higher when growth is driven by monopolists, one can easily verify that the equilibrium value of innovation is smaller. The reason is that larger investments in R&D reduce the lifelenght of new intermediates.

one for the leaders. It is immediate to verify that the optimal subsidies would be:

$$s^* = \frac{\eta}{\psi(1-\epsilon)+\eta\epsilon} \in (0,1), \quad s_M^* = \frac{\eta\epsilon}{\psi(1-\epsilon)+\eta\epsilon} \in (0, s^*) \tag{39}$$

This is in contrast with the model of R&D investment by monopolists due to an exogenous technological advantage by Segerstrom [2006], which delivers the optimality of a negative R&D subsidy. This may show the importance of endogenizing monopoly persistence rather than assuming it through exogenous technological cost advantages.

4 Strategic R&D Policy

In this section, following the litarature on strategic trade policy (see Krugman and Helpman, 1989; Spencer, and Brander, 1983) I will characterize the unilateral incentives of single countries to subsidize R&D to verify whether a decentralized adoption of R&D policies can implement Pareto improving policies.

To focus on the crucial issues, consider the decentralized equilibrium with Nash competition in the market for innovations, but imagine that country H can choose an R&D subsidy s_H to promote its own firms in the international competition for innovation κ_j for ant sector j. Hence, the expected profits of its domestic firms active in sector j are:

$$\Pi^H(k_j, s_H) = \frac{[\phi(\kappa_j) z_H(\kappa_j)]^\epsilon \mathbf{V}^M(\kappa_j+1) - (1-s_H) z_H(\kappa_j)}{r + p(\kappa_j)} - F(\kappa_j) \tag{40}$$

As we will see, whether country H subsidizes or not its firms, the aggregate variables like the growth rate and the world technological progress will not be affected, hence the only reason for which country H may want to subsidize its firms is profit-shifting. As usual in the theory of strategic trade policy, a country may undertake unilateral export promoting policies to increase net welfare in terms of expected profits net of the expected cost of the subsidies:

$$W(s_H) = \sum_{j=1}^{N} \left[\Pi^H(k_j, s_H) - \frac{s_H z_H(\kappa_j)}{r + p(\kappa_j)} \right] \tag{41}$$

The problem of optimal R&D subsidization is a particular case of a more general problem of optimal strategic export promotion investigated by Etro [2003] (and there applied to traditional export subsidies and competitive devaluations), whose results imply in our framework the optimality of a positive R&D subsidies. More generally Etro [2003], generalizing the results by Brander and Spencer (1985), shows that when the profit function of the subsidized domestic firm satisfies strategic complementarity, that is $\partial^2 \Pi^i / \partial z_i \partial z_f > 0$, and marginal profitability increasing in the subsidy, that is $\partial^2 \Pi^i / \partial z_i \partial s > 0$, as here for (40) with the value of innovation (11), 1) under barriers to entry there is always a unilateral incentive to tax the domestic firm, but 2) under free entry there is always a unilateral incentive to subsidize the domestic firm.[21]

4.1 The optimal unilateral R&D subsidy

Consider the equilibrium in a single market for competition given the subsidy s_H. The first order conditions for the optimal investments $z_H(k_j)$ maximizing (40) and $z_i(k_j)$ maximizing (8) and the free entry condition which pins down the number of active foreign firms fully characterize the equilibrium. While the last two conditions are the same as before and given by (13) and (14), the first order condition for the subsidized firm is:

$$\left[\epsilon\phi(\kappa_j)^{\epsilon} z_H(\kappa_j)^{\epsilon-1}\mathbf{V}^M(\kappa_j+1) - 1 + s_H\right][r + p(\kappa_j)] =$$

$$= \epsilon\phi(\kappa_j)^{\epsilon} z_H(\kappa_j)^{\epsilon-1}\left\{[\phi(\kappa_j)z_H(\kappa_j)]^{\epsilon}\mathbf{V}^M(\kappa_j+1) - (1-s_H)z_H(\kappa_j)\right\} \qquad (42)$$

This system provides investment by foreign firms and aggregate probability of innovation $p(\kappa_j)$ independently from the domestic subsidy, while the investment of the domestic firm, $z_H(\kappa_j, s_H)$, is increasing in the subsidy (as the number of foreign entrants is decreasing in it). This implies that we can rewrite domestic welfare as:

$$W(s_H) = \sum_{j=1}^{N} \frac{[\phi(\kappa_j)z_H(\kappa_j, s_H)]^{\epsilon}\mathbf{V}^M(\kappa_j+1) - z_H(\kappa_j, s_H)}{r + p(\kappa_j)} - NF(\kappa_j) \qquad (43)$$

which is maximized as long as the optimal subsidy s_H^* satisfies:

$$\epsilon\phi(\kappa_j)^{\epsilon} z_H(\kappa_j, s_H^*)^{\epsilon-1}\mathbf{V}^M(\kappa_j+1) = 1 \qquad (44)$$

The resulting investments:

$$z(\kappa_j) = \frac{\left\{\epsilon\phi(\kappa_j)\left[\mathbf{V}^M(\kappa_j+1) - F(\kappa_j)\right]\right\}^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)} < z_H(\kappa_j, s_H^*) = \frac{\left[\epsilon\phi(\kappa_j)\mathbf{V}^M(\kappa_j+1)\right]^{\frac{1}{1-\epsilon}}}{\phi(\kappa_j)}$$

can be compared with (30) to realize that the government subsidy puts the domestic firm in the position of a Stackelberg leader in its patent race, a well known result in the theory of strategic trade policy. Nevertheless, as long as

[21] One can verify that here (in equilibrium) we have:

$$\frac{\partial^2\Pi^H}{\partial z_H \partial z_f} = \frac{\epsilon\phi(\kappa_j)^{\epsilon} z_H(\kappa_j)^{\epsilon-1}\mathbf{V}^M(\kappa_j+1) - (1-s_H)}{[r+p(\kappa_j)]^2} > 0$$

$$\frac{\partial^2\Pi^H}{\partial z_H \partial s} = \frac{r + p(\kappa_j) - \epsilon\phi(\kappa_j)^{\epsilon} z_H(\kappa_j)^{\epsilon}}{[r+p(\kappa_j)]^2} > 0$$

hence the results by Etro (2003) apply.

the R&D subsidies are just provided for the single patent race and not for the future ones, the value any innovation is not affected and the equilibrium aggregate variables are unchanged compared to the basic decentralized equilibrium. Clearly, if a government could commit to subsidize forever R&D investors including firms that achieved the world leadership, the value of innovating for a domestic firm would be above the corresponding value for a foreign firm, which could induce endogenous investment by domestic patentholder in a way similar to the previous section. This could possibly enhance growth, but a complete analysis of this case is beyond the scope of this paper.

Let us go back to our optimal subsidy. Substituting (44) in the equilibrium system we can implicitly derive the optimal subsidy as:

$$s_H^* = \left[1 + \frac{[r + p(\kappa_j) - [\phi(\kappa_j)z_H(\kappa_j)]^\epsilon]\,\mathbf{V}^M(\kappa_j+1)}{[\phi(\kappa_j)z_H(\kappa_j)]^\epsilon \mathbf{V}^M(\kappa_j+1) - z_H(\kappa_j)}\right]^{-1} \in (0,1)$$

Finally, adopting our assumptions on the functional forms, we can explicitly obtain the optimal unilateral R&D subsidy for any domestic firm as:

$$s_H^* = \frac{\eta\,(1-\epsilon)}{\left(\frac{\mu-\eta}{\mu}\right)^{\frac{\epsilon}{1-\epsilon}}\left[\mu\,(1-\varepsilon)+\eta\epsilon\right]-\epsilon\eta} \in (0,1) \tag{45}$$

Unfortunately, these policy just shift profits from one country to another, crowding out investments by foreign firms in favour of the domestic subsidized firm, while aggregate growth is unaffected:

in the open economy context, independent R&D policies do not promote growth, suggesting a new and strong case for international R&D cooperation.

4.2 On the international decentralized Nash equilibrium

The natural question one may ask at the end of this discussion is whether a Nash equilibrium with positive symmetric subsidies can emerge when all countries can choose their R&D policy in a decentralized way. This is what happens in standard models of strategic trade policy with imperfect competition, where the Nash equilibrium is inefficient compared to the cooperative solution.[22]

First of all, it is easy to verify that the optimal subsidies are strategic substitutes in this model (one can rework the derivation of the optimal R&D subsidy when the other countries have already adopted a small subsidy s and verify that $\partial s_H^*(s)/\partial s < 0$: the intuition is that foreign R&D subsidization reduces the gains from domestic investment. Nevertheless, it is also easy to verify that, international free entry makes unfeasible a symmetric equilibrium where all countries adopt the same subsidy: if this was the case, all firms would expect

[22]Notice that in the standard static literature on export subsidies (as in Brander and Spencer, 1985) the Nash equilibrium implies excessive subsidization (since the optimal cooperative policy requires export taxes).

the same zero profits and the subsidization policies would just have a cost without a benefit.[23]

While asymmetric equilibria with some countries subsidizing their firms and other countries not doing it could exist, their characterization is beyond the scope of this paper. What matters here, is that, even in such a case, any equilibrium must be characterized by the free entry condition holding on firms without subsidies. Once again, this implies that any R&D policy by any of the other countries would not affect aggregate growth.

This discussion also emphasizes that R&D policy coordination can be quite hard to implement, since there are strong incentives to deviate and reduce R&D subsidies for every single country when the other are subdidizing R&D as well. As well, known, problems in achieving international coordination of policies are even greater when there is pervasive heterogeneity between countries and hence between their favourite policies.[24] Hence in the next section we look at a more realistic framework where trade barriers induce a genuine heterogeneity between trading countries.

5 Trade Barriers

The above model can be used for a wide range of investigations on other determinants of growth and the way policies affect the engine of growth. In this section I will show that in presence of trade frictions growth is driven by the largest economy and enhanced by its relative size and by openness.

In the general model of Section 2 we introducedtrade frictions thorugh a sort of iceberg cost tool for which a unit of intermediate good sent to a foregin country becomes d units at destination. Here I will focus on the case in which $d < 1$: in such a case, foreign demand is smaller than domestic demand for each intermediate good produced at home, and the expected discounted value of the profits with innovation k_j by a firm in country i becomes:

$$V_i(\kappa_j) = \frac{\mu \left(\frac{\alpha q^{\kappa_j \alpha}}{1+\mu} \right)^{\frac{1}{1-\alpha}} \left[A_i^{\frac{1}{1-\alpha}} L_i + d^{\frac{1}{1-\alpha}} \sum_{f \neq i} A_f^{\frac{1}{1-\alpha}} L_f \right]}{r + p\left(k_j\right)} \tag{46}$$

which is higher for larger economies.[25] As long as the productivity of the research efforts is the same in all countries, or higher in the larger ones, the

[23]Formally, in the plane (s, s_H^*) there is not a symmetric equilibrium with $s_H^*(s) = s$ because the best response function has a discontinuity (jumps to $s_H^*(s) = 0$ for any $s > \hat{s}$ with $s_H^*(\hat{s}) > \hat{s}$).

[24]See Alesina, Angeloni and Etro (2005).

[25]This specification is borrowed from Alesina and Barro [2002] who studied the relation between the size of countries and unions and globalization - see also Etro [2005]. Notice that in case of different growth rates of TFP and population across countries, we should take into account the growth rates as well to compare country'sizes.

endogenous allocation of R&D investment is always biased toward the larger country: under Nash competition in the market for innovations, only its firms will invest in R&D, while under Stackelberg competition incumbent firms from other countries may keep investing and retain the leadership, but they will loose it sooner or later in favour of firms from the largest country. Hence, for any initial allocation of the technological frontier, the engine of world growth is in the largest economy, which gradually conquers the technological leadership in all sectors through its innovative firms. Historically, a similar process realized during the XIX century with the Industrial Revolution in England and in the XX century when USA became first the largest world economy and then gradually conquered the technological leadership in most sectors; some observers would bet on China repeating this path during the XXI century.

The world economy must be characterized by a constant growth rate for all countries (differences would emerge introducing heterogeneity in TFP growth across countries). Growth increases in the relative size of the leading country

$$b = \max\left(L_f A_f^{1/(1-\alpha)}\right) / \left(\sum L_f A_f^{1/(1-\alpha)}\right)$$

For instance, when $\epsilon \to 1$ the growth rate, as a function of the degree of openness, boils down to:

$$g(d) = \frac{\frac{\mu}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left[b + d^{\frac{1}{1-\alpha}}(1-b)\right] - \rho}{\theta + (1-\alpha)/\alpha \ln q}$$

Even if this model does not exhibit absolute scale effects as in the last generation of endogenous growth models (since Jones [1995]), "relative scale effects" emerge: the larger is the leading economy compared to the rest of the world, the higher is the growth rate. A consequence of these relative scale effects is that the positive relation between openness and growth survives (see Barro and Sala-i-Martin [2004], on the related empirical evidence).

The equilibrium is characterized by intra-industry trade: even if the largest country has an absolute advantage in all sectors, it develops a comparative advantage in the intermediate goods sector and exports these goods and imports final goods in the long run.[26] Finally, notice that the world interest rate has to equate world savings and investment, hence savings from all the world finance investments in the leading country. This may help explaining the paradox of Lucas (1990) concerning why capital does not fly to poor countries, while often there is a flow of resources moving in the opposite direction. Summarizing,

in a open economy context with trade frictions, the largest country has a comparative advantage in the innovation sector and, in the long run, it leads alone the technological frontier, exports intermediate goods, imports final goods

[26] At each point in time, the trade surplus in intermediate goods, national savings and net capital inflows will have to be matched by a deficit in trade of final goods and by investments.

and attracts foreign capital to finance investment. Growth increases in the degree of openness and in the relative size of the largest country.

Such a stylized scenario appears in line with the growth experience of the last decades, which was characterized by 1) large R&D investments and high rate of technological progress in the US, 2) high US imports of final goods which allowed other countries to grow as well, exporting final goods to the US and importing American technology, and by 3) impressive capital flows toward the US financing its large current account deficits and turning United States into the largest debtor country in the world.

Also in this framework, countries would like to subsidize their firms, but in different ways. The leading country could do it with the direct purpose of enhancing growth toward the efficient level. Other advanced countries could do it just to promote investment by their firms and conquer a leadership with the associated profits in some sectors (their optimal subsidy would also change with their relative size) without affecting the growth rate. Finally, less advanced countries could not even obtain advantages from subsidies. Once again, there would be a case for international coordination of R&D policies, but now heterogenity between countries would create different incentives for different countries so as to complicate coordination even further. While a full analysis of world coordination is beyond the scope of this paper, we confirm the important result that when international growth is driven by endogenous technological progress there is a strong case for international R&D policy coordination.

6 Conclusions

This paper has developed an open economy Schumpeterian growth model where fully fledged patent races drive investments in innovation and, when the incumbent patentholders are leaders in the patent races, they also invest in R&D, in which case their monopolistic position is partially persistent and leads the growth process. The model provides new insights on the growth process. The equilibrium is characterized by a new form of dynamic inefficiency due to an inefficient bias toward too small firms in the market for innovation, and R&D subsidies are always part of the optimal innovation policy exactly because they help increasing the size of firms. However, even if the optimal unilateral strategic policy for each country requires positive subsidies, we have shown that decentralized R&D choices do not enhance growth. In presence of trade frictions, the largest country has a comparative advantage in the innovation sector and, in the long run, it leads alone the technological frontier, exports intermediate goods, imports final goods and attracts foreign capital to finance investment. Also in such a more general framework, we emphasized a strong case for international R&D policy coordination.

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Appendix

I will now provide further details on the model with Stackelberg leadership for patentholders in the R&D sector when $\epsilon \geq \hat{\epsilon}$, which means that the optimal strategy for the leader is pure entry deterrence (a more general result on entry deterrence in present of Stackelberg competition is in Etro, 2006a). To derive the equilibrium under this regime of complete persistence of monopoly, notice that the investment of the leader must be slightly above the level at which the free entry condition allows entry by just one follower. Such an investment allows the leader to be alone in the patent race and, adopting the usual guess for the value function (31), it implies:

$$r+p=\frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}(\psi-\eta)$$

Using the equilibrium expression for the growth rate of consumption (3) and income (37), this provides:

$$\psi=\eta+\frac{p\left[1+\frac{\theta\alpha\ln q}{1-\alpha}\right]+\rho}{\frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}}$$

which is a standard positive relation between the probability of innovation and the return from leadership. Since the leader is alone in the patent race, the probability of innovation is simply $p=[\phi(k_j)z_M(k_j)]^{\epsilon}$, which implies:

$$z_M(k_j)=p^{(1/\epsilon)}/\phi(k_j)$$

Now the Bellman equation expressing the value of leadership becomes:

$$\begin{aligned}\mathbf{V}^M(\kappa_j) &= \frac{[\phi(k_j)z_M(k_j)]^{\epsilon}\,\mathbf{V}^M(\kappa_j+1)-z_M(\kappa_j)}{r+p}+V(\kappa_j)-F(\kappa_j)= \\ &= \frac{X(\kappa_j)}{r+p}\left[q^{\frac{\alpha}{1-\alpha}}p\frac{\psi}{r+p}+\mu-(1+\mu)^{\frac{1}{1-\alpha}}q^{\frac{\alpha}{1-\alpha}}p^{\frac{1}{\epsilon}}-\eta q^{\frac{\alpha}{1-\alpha}}\right]\end{aligned}$$

which, using the guess value for $V^M(\kappa_j)$, provides:

$$\psi=\left[\frac{\mu-(1+\mu)^{\frac{1}{1-\alpha}}q^{\frac{\alpha}{1-\alpha}}p^{\frac{1}{\epsilon}}-\eta q^{\frac{\alpha}{1-\alpha}}}{\rho-\frac{(1-\theta)p\alpha\ln q}{1-\alpha}}\right]\left\{\rho+p\left[1+\theta\frac{\alpha\ln q}{1-\alpha}\right]\right\}$$

whose denominator must be positive under the transversality condition $\rho>(1-\theta)\,g$, which requires θ large enough. This implies a negative relationship between the probability of innovation and the return from leadership due to the entry deterrence constraint: the larger is the investment in R&D needed to deter entry, the smaller is the value of being a leader. The two conditions above define the equilibrium values for $\psi=\bar{\psi}$ and the aggregate probability of innovation and hence the interest rate, the effective discount rate and the growth rate:

$$g=\frac{\frac{\epsilon^{\epsilon}}{(1+\mu)^{\frac{\epsilon}{1-\alpha}}}\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon}(\bar{\psi}-\eta)-\rho}{\theta+(1-\alpha)/\alpha\ln q}$$

In general, the leader is investing just enough to deter entry, while it could marginally reduce its investment and allow entry by one follower, which would increase the aggregate probability of innovation, the effective discount rate, and hence also the growth rate (this implies that in the regime of entry deterrence, the equilibrium growth rate could be below the one emerging without leaderships). However, approaching constant marginal productivity, the return from leadership $\bar{\psi}$ tends to μ and:

$$g\lfloor_{\epsilon\to 1,\eta\to 0}=\frac{\mu/(1+\mu)^{\frac{1}{1-\alpha}}-\rho}{\theta+(1-\alpha)/\alpha\ln q}$$

which is the same growth rate as under Nash competition in the market for innovations: when $\epsilon = 1$ the incumbent monopolist is actually indifferent between investing in R&D and crowding out outsiders' investment or not investing at all (with no changes in the aggregate equilibrium variables).