Top-dog and the Lean and Hungry Look in Endogenous Entry

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Abstract

This paper examines the strategic commitment behavior of heterogeneous leaders in endogenous market structure. We demonstrate that each leader's optimal investment level is independent of the other leaders' characters. Furthermore, we show that a leader over- (resp. under-) invests when an investment increases (resp. decreases) the leader's marginal profitability. Such investment always makes leaders take aggressive strategies in competition relative to those in no-commitment case. Our results clarify that aggressiveness of leaders is a robust observation in endogenous market structure.

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1 Introduction

This paper examines the strategic commitment behavior of leaders in economic circumstances. In particular, we consider how market leaders determine their investment and how these decisions affect the outcome of the competition with endogenous entrants. For exogenous structure case, the implication of strategic commitment is systematically investigated by the seminal work of Fudenberg and Tirole (1984).[1] According to them, the optimal investment level is determined by two factors: (i) whether strategic substitutability holds or complementarity does, and (ii) whether an investment increases or decreases the leader's marginal profitability. Their main conclusions are as follows. If an investment increases a leader's marginal profitability, then the leader has an incentive to over- (resp. under-) invest with strategic substitutability (resp. complementarity). If an investment decreases the leader's marginal profitability, the leader has an incentive to under- (resp. over-) invest with strategic substitutability (resp. complementarity).[2]

Recently, Etro (2006) has shown that strategic commitment behavior drastically changes in endogenous market structure.[3] He assumes that there is one leader in a market, and proves that regardless of strategic substitutability or complementarity, the leader over- (resp. under-) invests when an investment increases (resp. decreases) the leader's marginal profitability. In addition, the leader always takes aggressive strategy in the competition: it produces larger quantity in the quantity setting competition, or it offers lower price in the price competition than the entrants.

The purpose of this paper is to provide further remarks on leaders' strategic investment in endogenous market structure. We generalize Etro's analysis in two points. First, there exists one leader in Etro's model while we consider multiple leaders in a market.[4] Second, Etro (2006) considers a special setting where a leader and followers choose the same output (or price) in no-commitment case, while we allow more general situations. We demonstrate that each leader's optimal investment level is independent of the other leaders' characters even if we consider heterogeneous leaders. Furthermore, we show that the leader over- (resp. under-) invests when an investment increases (resp. decreases) the leader's marginal profitability. Such an investment always makes each leader take aggressive strategy in the competition relative to that in no-commitment case. Our results clarify that aggressiveness of leaders is a robust observation in endogenous market structure, and that leaders' pre-commitment level is separately determined in endogenous market structure.

The rest of this paper is organized as follows. Section 2 provides our model and Section 3 presents our results. Section 4 concludes the paper. All proofs are relegated to the Appendix.

[1] Fudenberg and Tirole (1984) consider a duopoly model with one leader and one follower.

[2] See Bulow et al. (1985) for a detailed discussion of strategic substitutability and complementarity.

[3] For other work on strategic commitment in endogenous market structure, see also Okuno-Fujiwara and Suzumura (1993).

[4] Daughety (1990) examines Stackelberg model with multi-leaders. Ino and Matsumura (2009) extend Daughety's model in endogenous market structure.

2 The Model

Consider n of leaders and m of entrants in a market. Firm $\ell \in \{1, ..., n\}$ is a leader firm. Each leader firm ℓ chooses a strategic variable a_ℓ and an investment I_ℓ. The investment I_ℓ can be interpreted as a cost reducing investment, or advertisement and so forth. Firm $e \in \{n+1, \ldots, n+m\}$ is an entrant firm. They decide to enter the market as long as they get positive profits. If an entrant enters the market, it chooses a strategic variable a_e in the competition.

Strategic variables a_ℓ and a_e have several interpretations: it represents output level of firm ℓ and firm e in the case of quantity competition, price level of firm ℓ and firm e in the case of price competition, and also it represents quality level of firm ℓ and firm e in the case of quality competition.[5]

Let $i \in \{1, ..., n+m\}$. Define $b_i : \mathbb{R}_+^{n+m-1} \to \mathbb{R}_+$ by:

$$b_i = \sum_{j \in \{1, ..., i-1, i+1, ..., n+m\}} h(a_j),$$

where a function $h : \mathbb{R}_+ \to \mathbb{R}_+$ is differentiable and strictly increasing. The function b_i aggregates the strategic variables a_j other than firm i's one through the function h.

The variable I_ℓ is the strategic investment level that is chosen by each leader ℓ. Thus, the profit of leader $\ell \in \{1, ..., n\}$ is given by:

$$\pi^\ell = \Pi^\ell(a_\ell, b_\ell, I_\ell) - f(I_\ell) - F,$$

where $f(I_\ell)$ is the investment cost with $f'(I_\ell) > 0$ and $f''(I_\ell) > 0$, and $F > 0$ is a fixed cost.

Entrant firm $e \in \{n+1, \ldots, n+m\}$ has a profit function given by:

$$\pi = \Pi(a_e, b_e, I) - F,$$

where $F > 0$ is a fixed entry cost. For all entrant firms, the last argument I is given and constant.

Let Π_k^ℓ and Π_k denote derivative of k-th argument of Π^ℓ and Π respectively. In the same way, Π_{kj}^ℓ and Π_{kj} denote the second derivatives of Π^ℓ and Π respectively. As a standard assumption, we assume that the effect induced by the strategies of the other firms is negative on the profit of each firm, i.e., $\Pi_2^\ell < 0$ and $\Pi_2 < 0$. We also assume that the leader's investment is profit-enhancing i.e., $\Pi_3^\ell > 0$. Furthermore, the second-order condition is assumed to be satisfied.

For each $\ell \in \{1, ..., n\}$, if an investment of leader ℓ increases its marginal profitability ($\Pi_{13}^\ell > 0$), we will say that the leader's investment makes the leader *tough*; if an investment of leader ℓ decreases its marginal profitability ($\Pi_{13}^\ell < 0$), the leader's investment makes the leader *soft*.

Our general model can be applied to commonly used models of oligopolistic competition in quantities and in prices. Consider entant firm $e \in \{n+1, \ldots, n+m\}$. When we consider a quantity setting competition model, the strategy a_e represents the quantity produced by entrant firm e. The inverse demand for firm e is defined as $P^e = P(a_e, \sum_{j \neq e} h(a_j))$. With the production cost function $C(a_e)$, we can specify

[5] Etro (2007) gives several models with these interpretations.

the profit function of entrant firm e given as $\Pi(a_e, b_e) = P(a_e, b_e)a_e - C(a_e)$. Now, we consider a price setting competition model where p_e is the price of firm e. The demand for firm e can be written as $D^e = D(p_e, \sum_{j \neq e} g(p_j))$ with $\partial D/\partial p_e < 0$, $\partial D/\partial p_j > 0$, and $g'(\cdot) < 0$. Set $a_e \equiv 1/p_e$ and $h(a_e) = g(1/a_e)$, and assume that the constant marginal cost is c, then we rewrite the profit function of entrant firm e as $\Pi(a_e, b_e) = D(1/a_e, b_e)(1/a_e - c)$. We can easily confirm that these two models are consistent with our general model by satisfying all the assumptions. We can formulate the profit function of each leader firm $\ell \in \{1, ..., n\}$ in both models of the quantity setting competition and the price setting competition in a similar way.

In Section 3, we consider two cases with respect to the timing of leader's investment: pre-commitment case and no-commitment case. In pre-commitment case, each leader can invest before the competition in the market. On the other hand, in no-commitment case, each leader chooses its investment level and strategic variable in the competition at the same time. At the end of Section 3, we compare the result of these two cases and consider the implication.

3 Result

Pre-commitment Case In this case, the game has three stages. At the first stage, each leader $\ell \in \{1, ..., n\}$ independently chooses its strategic commitment variable I_ℓ. At the second stage, potential entrants decide whether to enter the market or not. At the third stage, all firms choose their strategic variables independently: each leader ℓ chooses a_ℓ and each entrant e chooses a_e. In this case, strategic variable a_ℓ and a_e are chosen after observing investment profile $(I_1, ..., I_n)$ of leaders as strategic commitments.

We will look at subgame perfect Nash equilibrium by solving this three-stage game backwards. We assume that strictly positive number of entrants enter in the equilibrium, i.e., $m > 0$. At the third stage, given an investment profile $(I_1, ..., I_n)$ of leaders, competition takes place. We have the first-order conditions of leaders and entrants as follows:

$$\Pi_1^\ell(a_\ell, b_\ell, I_\ell) = 0 \text{ for } \ell \in \{1, ..., n\},$$
$$\Pi_1(a_e, b_e, I) = 0 \text{ for } e \in \{n+1, ..., n+m\}.$$

As we mentioned in Section 2, we assume that the second-order conditions are satisfied.

In what follows, we focus on symmetric equilibrium where all entrant firms take the same strategies, i.e., $a_e = a_E$ for all $e \in \{n+1, \ldots, n+m\}$. Then, we can rewrite b_e of each entrant e as $b_e = b_E = \sum_{k=1}^{n} h(a_k) + (m-1)h(a_E)$. We also have $b_\ell = b_E + h(a_E) - h(a_\ell)$ for each leader firm ℓ.

At the second stage, entrants enter as long as they can get positive profit. Now we have equilibrium conditions in the second- and third-stage as follows:

$$\Pi_1^\ell(a_\ell, b_E + h(a_E) - h(a_\ell), I_\ell) = 0 \text{ for } \ell \in \{1, ..., n\}, \tag{1}$$

$$\Pi_1(a_E, b_E, I) = 0, \tag{2}$$

$$\Pi(a_E, b_E, I) - F = 0. \tag{3}$$

Equation (3) is the zero-profit condition of entrants. Then, we have the following lemma.

Lemma 1 *Both a_E and b_E are independent of any investment profiles $(I_1, ..., I_n)$ of leaders.*

Note that instead of each entrant's strategy, the strategy a_ℓ of each leader ℓ is dependent on its own investment I_ℓ as induced by equation (1). Let a_E^*, b_E^*, $(a_1^*, ..., a_n^*)$, and $(I_1^*, ..., I_n^*)$ denote the equilibrium values for a_E, b_E, $(a_1, ..., a_n)$ and $(I_1, ..., I_n)$.

Then we consider the first stage of this game. Each leader ℓ chooses investment I_ℓ independently to maximize its profit as follows:

$$\max_{I_\ell} \left[\Pi^\ell(a_\ell, b_E + h(a_E) - h(a_\ell), I_\ell) - f(I_\ell) - F\right].$$

Then the first-order condition of each leader ℓ at the first stage is:

$$\Pi_3^\ell - \frac{da_\ell}{dI_\ell}\Pi_2^\ell h'(a_\ell) - f'(I_\ell) = 0.$$

By differentiating equation (1) with respect to I_ℓ, we can rewrite the equilibrium condition of leader firm ℓ:

$$\Pi_3^\ell + \frac{h'(a_\ell^*)\Pi_2^\ell\Pi_{13}^\ell}{\Pi_{11}^\ell - h'(a_\ell^*)\Pi_{12}^\ell} = f'(I_\ell^*). \tag{4}$$

The system of equations (1)–(4) determine a_E^*, b_E^*, $(a_1^*, ..., a_n^*)$, and $(I_1^*, ..., I_n^*)$.

According to equation (4), each leader ℓ decides its investment level by comparing its effect on the profit and on the cost. The effect on the profit is consisted with the direct effect (Π_3^ℓ) and the strategic effect ($h'(a_\ell)\Pi_2^\ell\Pi_{13}^\ell/(\Pi_{11}^\ell - h'(a_\ell)\Pi_{12}^\ell)$). By a strategic commitment, the leader affects the behavior of the followers, that has the marginal effect on each leader's profit $h'(a_\ell)\Pi_2^\ell\Pi_{13}^\ell/(\Pi_{11}^\ell - h'(a_\ell)\Pi_{12}^\ell)$. Moreover, note that the optimal investment is determined independently of its impact on the behavior of the other leaders. Even if there exist heterogeneous leaders, each leader does not take into account the effect of its investment on the other leaders.

No-commitment Case We consider the case where the leader firms cannot invest before competition. The game is consisted with two stages. At the first stage, entrants decide whether to enter the market or not. At the second stage, each leader's investment level is determined and competition occurs: each leader firm ℓ chooses a_ℓ and I_ℓ, and each entrant firm e chooses a_e independently.

We will solve this two-stage game backwards. At the second stage, all firms compete with their strategic variables a_ℓ and a_e. In addition, each leader firm chooses I_ℓ independently. We have the first-order conditions of leaders and entrants as follows[6]:

$$\Pi_1^\ell(a_\ell, b_\ell, I_\ell) = 0 \text{ for } \ell \in \{1, ..., n\},$$
$$\Pi_3^\ell(a_\ell, b_\ell, I_\ell) - f'(I_\ell) = 0 \text{ for } \ell \in \{1, ..., n\},$$
$$\Pi_1(a_e, b_e, I) = 0 \text{ for } e \in \{n+1, ..., n+m\}.$$

Hereafter, we focus on symmetric equilibrium where all entrants have the same strategic variable. Thus, we have $a_e = a_E$ for all $e \in \{n+1, \dots, n+m\}$. Then, we can rewrite b_e of each entrant e as $b_e = b_E = \sum_{k=1}^{n} h(a_k) + (m-1)h(a_E)$. Moreover, we have that $b_\ell = b_E + h(a_E) - h(a_\ell)$ for each leader firm ℓ.

At the first stage, entrants enter the market as long as they obtain positive profits. Then the zero-profit condition of the entrants holds. Let $\bar{a}_E$, $\bar{b}_E$, $(\bar{a}_1, ..., \bar{a}_n)$, and $(\bar{I}_1, ..., \bar{I}_n)$ denote the equilibrium values of a_E, b_E, $(a_1, ..., a_n)$, and $(I_1, ..., I_n)$ respectively. Hence, we have the equilibrium conditions as follows:

$$\Pi_1^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), \bar{I}_\ell) = 0 \text{ for } \ell \in \{1, ..., n\}, \tag{5}$$
$$\Pi_3^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), \bar{I}_\ell) - f'(\bar{I}_\ell) = 0 \text{ for } \ell \in \{1, ..., n\}, \tag{6}$$
$$\Pi_1(\bar{a}_E, \bar{b}_E, I) = 0, \tag{7}$$
$$\Pi(\bar{a}_E, \bar{b}_E, I) - F = 0. \tag{8}$$

Equation (8) is the zero-profit condition of entrants. The system of equations (5)–(8) provides equilibrium values $\bar{a}_E$, $\bar{b}_E$, $(\bar{a}_1, ..., \bar{a}_n)$, and $(\bar{I}_1, ..., \bar{I}_n)$.

Obviously, as Lemma 1 in pre-commitment case, equations (7) and (8) induce both a_E and b_E in no-commitment case are independent of any investment profiles $(I_1, ..., I_n)$ of leaders. Furthermore, it is clear that conditions of entrants in pre-commitment case (equations (2) and (3)) and no-commitment case (equations (7) and (8)) are exactly the same. Hence, we have that $\bar{a}_E = a_E^*$ and $\bar{b}_E = b_E^*$ in the equilibrium.

By equation (6), we can easily see that each leader's investment level is determined by only the direct effect of investment. It means that if leaders cannot commit their investment level in advance of competition, each leader's investment does not strategically affect the behavior of entrants.

Comparison Now, we compare the equilibrium outcome in pre-commitment case and no-commitment case. For the further discussion, let us define the terminology. For each $\ell \in \{1, ..., n\}$, if $I_\ell > \bar{I}_\ell$ (resp. $I_\ell < \bar{I}_\ell$), leader ℓ *over-invests* (resp. *under-invests*). That is, if leader ℓ invests more (resp. less) in pre-commitment case relative to that in no-commitment case, we call it over-investment (resp. under-investment). Over-investment (resp. under-investment) occurs through positive (resp. negative) strategic

[6] We assume that the second-order conditions are satisfied.

effect of the investment in pre-commitment case. Moreover, a strategy a_i is *aggressive* relative to a strategy a_j, when $a_i > a_j$. [7]

We present our main result in the following proposition.

Proposition 1 *(i) For all $\ell \in \{1, ..., n\}$, if $\Pi_{13}^{\ell} > 0$ (resp. $\Pi_{13}^{\ell} < 0$), then leader ℓ over-invests (resp. under-invests). (ii) In pre-commitment case, every leader's strategy is aggressive relative to that in no-commitment case.*

Proposition 1 implies that the investment decision of leader ℓ is only dependent on whether its investment makes leader ℓ tough or soft, and it is independent of the other leaders' characters. In addition, regardless of whether an investment makes a leader tough or soft, the investment makes each leader's strategy aggressive in pre-commitment case relative to that in no-commitment case.

According to the taxonomy of Fudenberg and Tirole (1984), under endogenous entry, each leader employs only "Top-dog" strategy (over-investment when an investment that makes a leader tough) or "Lean and hungry look" (under-investment when an investment that makes a leader soft) strategy. "Puppy-dog" (under-investment when an investment that makes a leader tough) and "Fat-cat" (over-investment when an investment that makes a leader soft) never appear.

Next, we consider the special case where each leader firm $\ell \in \{1, ..., n\}$ has the identical profit function normalized to induce $\bar{I}_\ell = I$ and $\bar{a}_\ell = \bar{a}_E$ in the equilibrium in no-commitment case. By this normalization, each identical leader independently chooses the investment level I that results in a symmetric equilibrium strategy in the competition i.e., $\bar{a}_\ell = \bar{a}_E$ for leader firm $\ell \in \{1, ..., n\}$ in no-commitment case. Then we have the following result.

Corollary 1 *Suppose that all leaders have the same profit function, and $\bar{I}_\ell = I$ and $\bar{a}_\ell = \bar{a}_E$ for every leader $\ell \in \{1, ..., n\}$. Then, every leader's strategy is aggressive relative to follower's.*

Consequently, Corollary 1 states that Proposition 1 of Etro (2006) is the result in the special case of our multi-leader model where there exists only one leader with a certain condition on the profit function.

4 Concluding Remarks

In this paper, we have examined heterogeneous leaders' strategic investments in the market under endogenous entry. We have shown that in pre-commitment case, each leader over- (resp. under-) invests relative to in no-commitment case when an investment increases (resp. decreases) the leader's marginal

[7] This definition is due to Etro (2006). He introduces a companion concept, accomodation: a strategy a_i is *accommodating* relative to a strategy a_j, when $a_i < a_j$.

profitability. Furthermore, we found that each leader's investment level is independent of the other leaders' characters even if leaders are heterogeneous. In addition, such investment decision makes each leader aggressive relative to its strategy in no-commitment case in the competition.

For example, if we consider a cost reducing investment executed before competition, and a standard Cournot type competition, each leader firm invests more and produces more than it does in no-commitment case. As the result of a cost-reducing investment, each leader produces more goods at a lower marginal cost. That makes the leader's profit increase.

Finally, our results clarify that aggressiveness of leaders is a robust observation in endogenous market structure. If we assume that there exists only one leader and a special setting where a leader and followers choose the same output (or price) in no-commitment case, we can replicate the result of Etro (2006): each leader always takes aggressive strategy relative to those of entrants.

Appendix

Proof of Lemma 1 We rewrite equations (2) and (3) as follows:

$$\Pi_1(a_E, b_E, I) = 0,$$
$$\Pi(a_E, b_E, I) - F = 0.$$

Then we have the system of two simultaneous equations with two variables, a_E and b_E. It is clear that any I_ℓ does not appear in both two equations. Thus these two conditions pin down a_E and b_E independently of any investment profile of leaders $(I_1, ..., I_n)$. ■

Proof of Proposition 1 (i) Take any $\ell \in \{1, ..., n\}$. From Lemma 1, both a_E^* and b_E^* is independent of $(I_1, ..., I_n)$. Moreover, we have $a_E^* = \bar{a}_E$ and $b_E^* = \bar{b}_E$. For simplicity, we write them as a_E and b_E, respectively. We now consider how the equilibrium value of strategic variable a_ℓ of the leader is determined given $(I_1, ..., I_n)$. In both pre-commitment case and no-commitment case, the equilibrium value a_ℓ satisfies the following equation:

$$\Pi_1^\ell(a_\ell, b_E + h(a_E) - h(a_\ell), I_\ell) = 0.$$

Thus, the equilibrium value of strategic variable a_ℓ of the leader is a function of I_ℓ. Note that a_ℓ does not depend on I_k with $k \neq \ell$. We write it as $a_\ell(I_\ell)$. Let $b_\ell(I_\ell) = b_E + h(a_E) - h(a_\ell(I_\ell))$. From equation (6), the equilibrium investment $\bar{I}_\ell$ in no-commitment case satisfies the following equation:

$$\Pi_3^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell) - f'(\bar{I}_\ell) = 0. \tag{9}$$

Evaluating the derivative of the leader's profit at the first stage of pre-commitment case at $\bar{I}_\ell$, we obtain the following equation:

$$\Pi_3^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell) + \frac{h'(a_\ell(\bar{I}_\ell))\Pi_2^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)\Pi_{13}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)}{\Pi_{11}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell) - h'(a_\ell(\bar{I}_\ell))\Pi_{12}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)} - f'(\bar{I}_\ell). \tag{10}$$

Substituting (9) into (10), it becomes

$$\frac{h'(a_\ell(\bar{I}_\ell))\Pi_2^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)\Pi_{13}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)}{\Pi_{11}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell) - h'(a_\ell(\bar{I}_\ell))\Pi_{12}^\ell(a_\ell(\bar{I}_\ell), b_\ell(\bar{I}_\ell), \bar{I}_\ell)}. \tag{11}$$

Since we assume that $h'(a_\ell) > 0$, $\Pi_2^\ell < 0$, and $\Pi_{11}^\ell - h'(a_\ell)\Pi_{12}^\ell < 0$, the sign of (11) is exactly the sign of Π_{13}^ℓ. If $\Pi_{13}^\ell > 0$, then (11) is positive, and it follows that $I_\ell^* > \bar{I}_\ell$ from concavity of the profit function in I_ℓ. If $\Pi_{13}^\ell < 0$, then (11) is negative, and it follows that $I_\ell^* < \bar{I}_\ell$ from concavity of the profit function in I_ℓ. The proof of (i) is completed.

(ii) Take any $\ell \in \{1, ..., n\}$. We first show the following claim:

$$\Pi_1^\ell(a_\ell, b_\ell, I_\ell^*) > \Pi_1^\ell(a_\ell, b_\ell, \bar{I}_\ell) \text{ for all } a_\ell \text{ and } b_\ell. \tag{12}$$

If $\Pi_{13}^\ell > 0$, we have $I_\ell^* > \bar{I}_\ell$ by (i), and thus (12) must hold. On the other hand, if $\Pi_{13}^\ell < 0$, we have $I_\ell^* < \bar{I}_\ell$ by (i), and thus (12) must holds as well. Then, the claim is proved.

Therefore, we have

$$\Pi_1^\ell(\bar{a}_\ell, \bar{b}_\ell, I_\ell^*) > \Pi_1^\ell(\bar{a}_\ell, \bar{b}_\ell, \bar{I}_\ell).$$

Since $\bar{b}_\ell = \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell)$ this implies that:

$$\Pi_1^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), I_\ell^*) > \Pi_1^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), \bar{I}_\ell). \tag{13}$$

Since we have $a_E^* = \bar{a}_E$ and $b_E^* = \bar{b}_E$, we can rewrite (13) as follows:

$$\Pi_1^\ell(\bar{a}_\ell, b_E^* + h(a_E^*) - h(\bar{a}_\ell), I_\ell^*) > \Pi_1^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), \bar{I}_\ell). \tag{14}$$

The first-order condition with respect to a_ℓ of the leader firm in no-commitment case is as follows:

$$\Pi_1^\ell(\bar{a}_\ell, \bar{b}_E + h(\bar{a}_E) - h(\bar{a}_\ell), \bar{I}_\ell) = 0. \tag{15}$$

Hence, we obtain

$$\Pi_1^\ell(\bar{a}_\ell, b_E^* + h(a_E^*) - h(\bar{a}_\ell), I_\ell^*) > 0. \tag{16}$$

Since we assume that $\Pi_{11}^\ell - h'(a_\ell)\Pi_{12}^\ell < 0$, it follows that $a_\ell^* > \bar{a}_\ell$. ■

Proof of Corollary 1 In pre-commitment case, we always have $a_\ell^* > \bar{a}_\ell$, for all $\ell \in \{1, ..., n\}$ from Proposition 1. By assumption, $\bar{a}_\ell = \bar{a}_E$ for all $\ell \in \{1, ..., n\}$. We also have that $a_E^* = \bar{a}_E$ from equations (2), (3) and (7), (8). Thus we have $a_\ell^* > \bar{a}_\ell = \bar{a}_E = a_E^*$. ■

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