Endogenous Market Structures and International Trade Theory

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Abstract

I apply the endogenous market structures approach to international trade. When markets are characterized by strategic interactions and endogenous entry, opening up to trade decreases the markups, reduces the local number of firms and increases their production, with a positive competition effect on welfare. This is shown in traditional models with linear and isoelastic demand under competition in quantities and prices, and in a 2x2x2 model which nests the Heckscher-Ohlin model, the Krugman model and the Brander-Markusen model. Finally, I characterize the optimal import tariff for a domestic market and the optimal production subsity for an integrated market in the presence of endogenous entry of international firms.

Key words: Endogenous market structures, gains from trade, import tariff, production subsidy
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The two main models of international trade, the Heckscher-Ohlin model of interindustry trade based on perfect competition and constant returns to scale and the Krugman model of intraindustry trade based on monopolistic competition and increasing returns to scale, represent competing theories that can be fully integrated in a unitary view (see Dixit and Norman, 1980). Not the same can be said for the theories of imperfect competition with strategic interactions, whose origin goes back at least to the works on competition in quantities between firms producing homogenous goods by Brander (1981) and Markusen (1981) and to the subsequent literature on strategic trade policy (Brander and Krugman, 1983; Brander and Spencer, 1985; Hortsmann and Markusen, 1986). With few exceptions, the literature based on what we may label as the Brander-Markusen approach has been focused on special examples of Cournot and Bertrand competition in partial equilibrium oligopolies, generating a variety of models that emphasize the price-reducing effect of trade but that can be hardly integrated within traditional trade theories and generate different policy implications depending on the forms of competition and on the number of firms. The lack of a systematic analysis of imperfect competition within trade theory has induced trade economists (as Neary, 2010) to talk about the existence of "two and a half theories of trade" rather than three competing theories. The aim of this article is to contribute to the development of a theory of international trade with markets characterized by imperfect competition, that is with strategic interactions in the choice of production or pricing, between a limited but endogenous number of firms. These two elements, strategic interactions and endogenous entry, define what we call the endogenous market structure both under autarchy and under trade.[1] They allow us to characterize the sources of the gains from trade, comparing them with those emerging under perfect competition or monopolistic competition, and to study the optimal trade policy for the domestic firms in this setup.[2]

We develop a rather general model of imperfect competition (including general forms of competition in quantities and prices) between an endogenous num-

[1]Horstmann and Markusen (1992) refer to endogenous market structures, but they simply focus on the entry decisions of two multinationals in two segmented national markets with competition in quantities and fixed costs of entry at the firm and plant level, characterizing how technological conditions and trade policy induce switching between equilibria with a single firm or with two firms. Bhattacharjea (1995) also refers to endogenous market structures in his analysis of trade policy, but investigates free entry equilibria for a given tariff rather than characterizing the optimal commitment to a tariff in the presence of free entry.

[2]The analysis of endogenous market structures in industrial organization has a wide tradition (see Sutton, 1991) and has been recently used to analyze multiple issues, including the role of strategic investments (Etro, 2006) and exogenous shocks (Anderson *et al.*, 2010), incentive and financial contracts (Etro, 2010, 2011b), growth (Peretto, 1996), business cycles (Devereux, Head and Lapham, 1996; Etro and Colciago, 2010) and more.

ber of firms and show that opening up to free trade induces always welfare gains due to the change in the market structure. First of all, trade integration strengthens competition and induces a generalized reduction of the markups, which is a primary source of gains from trade. As a second impact, the competition effect creates a negative feedback on profitability which crowds out the expansion of the number of produced varieties and leads to business destruction at the local level: this reduces (or eliminates in case of homogenous goods) the traditional gains from variety of the Krugman model with monopolistic competition. As a third impact, after trade integration the equilibrium market structure is characterized by a larger production of each surviving firm: this increase in concentration is welfare enhancing because reduces the expenditure in fixed costs, which creates another source of gains from trade. To exemplify these general results, we solve for the endogenous market structure in the traditional cases of a linear demand and isoelastic demand *à la* Dixit-Stiglitz - in the last case under both competition in quantities and in prices. Finally, we confirm our results in a 2x2x2 model where one sector is perfectly competitive and the other one is characterized by endogenous market structures with alternative forms of competition and product differentiation. This model nests the neoclassical Heckscher-Ohlin model, the Krugman model extended to general equilibrium (as in Helpman and Krugman, 1985) and the Brander-Markusen model with homogenous goods extended to general equilibrium (as in Lahiri and Ono, 1995), and it delivers simultaneously three sources of gains from trade: gains from comparative advantage due to differences in factor endowments, gains from variety due to the increase in product differentiation and gains from competition due to the reduction of markups.

We also point out that the endogenous market structures are inefficient under free trade because of imperfect competition and inefficient entry (see Mankiw and Whinston, 1986), which opens the door to new roles for policy. Trade policy in the presence of endogenous entry has been already considered in special cases as monopolistic competition (Helpman and Krugman, 1989) and Cournot competition with two goods (produced respectively by domestic and foreign firms) in a domestic market (Venables, 1985) and in an integrated market (Horstmann and Markusen, 1986). All these models have focused on the long run impact of policies, with endogenous entry of international and domestic firms. Since also the domestic firms end up with zero profits, any profit shifting rationale for protectionism disappears, and the optimal policy is only aimed at improving the terms of trade, exactly as in the neoclassical context: therefore the optimal policy requires an import tariff for a domestic market and a production tax on the exportable good for an integrated market. Contrary to these models, we focus on trade policies introduced to protect domestic firms from the endogenous entry of international firms. This case is important in the short/medium run when international firms may be already active abroad, their entry/exit in the domestic or integrated market is faster than the creation of new domestic firms, and trade policy is aimed at shifting profits toward the existing domestic firms.

Within our general model, we show that import tariffs increase prices and reduce foreign entry, with a negative impact on consumer surplus, and they shift profits toward the domestic firm, but production taxes and subsidies do not affect the domestic consumers surplus in any way. This allows us to provide a rather general characterization of the optimal trade policy for markets with endogenous structures and to exemplify it within our models with linear and isoelastic demand. The optimal policy for an integrated market requires always a positive subsidy to domestic production, which is also prohibitive with competition in quantities, homogenous goods and constant marginal costs. The optimal policy for the domestic market remains in line with what emerges in the previous literature: with competition in quantities, homogenous goods and constant marginal costs, the optimal subsidy to domestic production is again prohibitive, but in its absence, the optimal import tariff is positive if and only if the elasticity of the slope of the inverse demand is small enough.

The analysis of trade theory and policy in the presence of endogenous market structures has attracted attention recently. Foreign direct investment has been studied by De Santis and Stahler (2004) who endogenize the number of national and multinational firms active in two countries, and by Markusen and Stähler (2010) who investigate the impact of greenfield investments by multinationals and cross-border acquisitions in markets with endogenous entry. Also the celebrated model by Melitz (2003) with free entry of heterogenus firms has been extended to strategic interactions by van Long *et al.* (2009), but welfare and policy issues have not been addressed yet. Finally, our companion paper (Etro, 2011a) analyzes strategic export promotion in a foreign segmented market with endogenous market structures, characterizing the role of export subsidies, R&D subsidies and competitive devaluations, and focusing on the Nash equilibrium policies adopted simultaneously by multiple countries.

The paper is organized as follows. Section 1 introduces a general model of international endogenous market structures. Section 2 develops two examples based on linear and isoelastic demand, and characterize the general equilibrium of a 2x2x2 model. Section 3 is about trade policy for a domestic market. Section 4 is about trade policy for an integrated market. Section 5 concludes.

1 A general model

In this section we analyze endogenous market structures that belong to a general class introduced in Etro (2006) and extended to an international context. Consider a market in a closed economy whose size can be measured with the number of identical consumers L, and where every firm bears a fixed entry cost F. Assume that each active firm chooses a strategy $x(i)$ delivering the net profit function:

$$\pi_i = L\Pi\left[x(i), \beta_i, \mu_i\right] - F \tag{1}$$

where the gross profits per consumer $\Pi\left[x(i), \beta_i, \mu_i\right]$ depend on the strategy $x(i)$, on the aggregate statistic of the strategies of the other firms $\beta_i = \sum_{j \neq i} h(x(j))$ for a positive and increasing function $h(x)$, and on the exogenous parameter μ_i, which may be a policy variable (or a trade cost). We assume that profits are quasiconcave in $x(i)$ and decreasing in β_i ($\Pi_2 < 0$), but we allow for strategic substitutability ($\Pi_{12} < 0$) or complementarity ($\Pi_{12} > 0$). Under competition in quantities $x(i)$ is the output level of firm i, as in a model with linear, isoelastic or exponential inverse demand (microfounded respectively under quadratic, Dixit-Stiglitz or exponential preferences), or with an inverse demand $p(x(i), \sum_{j \neq i} h(x(j)))$ decreasing in both arguments. Under competition in prices, a model with a direct demand as $D(p(i), \sum_{j \neq i} h(1/p(j))$, decreasing in both arguments, is nested in our specification (our notation can be used with $x(i)$ as the inverse of the price level of firm i): this is the case of demand functions derived from Dixit-Stiglitz preferences, from the Logit model or from any constant expenditure model in which demand depends on an aggregate price index.

In the absence of firm specific heterogeneity ($\mu_i = \mu$ for any i), a general characterization of Nash equilibria with endogenous entry is straightforward. In a symmetric equilibrium, the first order condition and the endogenous entry condition define the equilibrium strategies x and number of firms n as follows:

$$\Pi_1[x, (n-1)h(x), \mu] = 0 \text{ and } \Pi[x, (n-1)h(x), \mu] = \left(\frac{L}{F}\right)^{-1} \tag{2}$$

under the assumption $\Pi_{11} < \Pi_{12}h'(x)$, which can be interpreted as a stability condition.[3]

[3]The slope of the best response function $\Pi_1[x(i), \beta_i, \mu] = 0$ can be derived as

$$\frac{dx(i)}{dx(j)} = -\frac{h'(x(j))\Pi_{12}[x(i), \beta_i, \mu]}{\Pi_{11}[x(i), \beta_i, \mu]}$$

In a symmetric equilibrium the best response function crosses the equilibrium condition $x(i) =$

It can be easily verified that an increase in the ratio between size of the market and fixed cost has ambiguous effects on x (which increases under strategic substitutability and decreases under strategic complementarity), but increases always the number of firms n. Moreover, as we will see in Section 2, in standard models it also strengthens competition, expands individual production, reduces prices and increases less than proportionally the number of firms. Contrary to what happens in the Krugman model, this is a crucial effect that we expect from opening up to trade with other countries to create a larger integrated market of size L^W: integration tends to reduce the international mark ups and the number of firms in each country, while it expands the output per firm and increases less than proportionally the total number of goods produced for the integrated market. In Section 2 we will find out the same phenomena after extending the model to a general equilibrium framework with two countries, two inputs and two goods, one of which is produced under imperfect competition.

Let us assume, as it holds in a wide class of models nested in our specification,[4] that the consumer surplus of the representative agent U is an increasing function of the aggregate statistic $\sum_{j=1}^{n} h(x(j))$. For instance, under competition in quantities consumer surplus depends on total production when the goods are homogenous and on a weighted sum of the production levels with standard forms of product differentiation (as those derived from isoelastic preferences), and under competition in prices with standard demand functions consumer surplus is decreasing in a properly defined price index. In a symmetric equilibrium, total welfare corresponds to the sum of consumer surplus of all agents and total profits:

$$W = LU\left[nh(x)\right] + n\left\{L\Pi[x,(n-1)h(x),\mu] - F\right\} \tag{3}$$

where the second term is zero when endogenous entry dissipates all profits. Therefore, the impact of the increase in the integrated market size L^W on welfare can be obtained differentiating (3) and using the total differentiation of the system (2) as follows:[5]

$$\frac{dW}{dL^W} = \frac{LU'\left[nh(x)\right]\Pi\left[\Pi_{12}h'(x) - \Pi_{11}\right]}{L^W \Pi_{11}\Pi_2}$$

$x(j)$. For this to be a unique equilibrium we need the slope to be smaller than 1 at the intersection, and for this to be a stable equilibrium (such that optimal dynamic adjustment of firm i starting from an out-of-equilibrium strategy of firm j leads back to the symmetric equilibrium) we also need the slope to be larger than -1. This amounts to the condition in the text.

[4]See for instance Anderson *et al.* (2010) or Etro (2011a,b).

[5]Notice that $U\left[nh(x)\right] = U[\beta + h(x)]$, whose differentiation with respect to the size of the market is $dU = \left[d\beta + h'(x)dx\right]U'$. Using the total differentiation of (2) provides $d\beta = -(\Pi/L\Pi_2)dL > 0$ and $dx = -(\Pi_{12}/\Pi_{11})d\beta$, which leads to the formula in the text.

which is always positive under our stability condition. This proves:

THEOREM 1. **Under endogenous market structures with competition in quantities or in prices, opening up to free trade is always welfare improving.**

While the Krugman model was generating gains from trade only through the benefits of an increase in the number of varieties consumed, here these benefits are less important (or absent when goods are perfect substitutes) and new gains from competition emerge: the higher number of firms due to market integration reduces the prices and increases the consumption of each variety. Nevertheless, endogenous market structures are characterized by an inefficient allocation of production both under autarchy and under free trade - to verify this, simply compare the equilibrium conditions with those for the maximization of (3) with respect to n and x. This opens the door to a new role for policy and trade policy in particular.

Our simple model allows us to evaluate the impact of trade policy interventions or structural changes for domestic, foreign and integrated markets. Let us consider a general market with a single domestic firm whose profitability is affected by a specific parameter μ, and $n-1$ foreign firms possibly affected by a common parameter μ^*. Let us denote with an asterisk the variables and the profit functions of the foreign firms. In the absence of prohibitive trade policies, the equilibrium strategies x and x^*, and the endogenous number of active firms n must satisfy the optimality and endogenous entry conditions:

$$\Pi_1[x, (n-1)h(x^*), \mu] = 0 \tag{4}$$

$$\Pi_1^*[x^*, (n-2)h(x^*) + h(x), \mu^*] = 0 \tag{5}$$

$$\Pi^*[x^*, (n-2)h(x^*) + h(x), \mu^*] = \left(\frac{L}{F}\right)^{-1} \tag{6}$$

It follows (from the last two conditions) that the strategy of the foreign firms x^* depends on μ^*, but not on μ. Most important, even the aggregate statistic perceived by the foreign firms, $\beta^* = (n-2)h(x^*) + h(x)$, depends on μ^* but not on μ. As a consequence also the consumer surplus is independent from μ, because the aggregate statistic $\sum_{j=1}^{n} h(x(j)) = \beta^* + h(x^*)$ depends on μ^*, but does not change with μ. On the other hand, the strategy of the domestic firm x and the number of entrants:

$$n = 2 + \frac{\beta^* - h(x)}{h(x^*)}$$

depend on both the policy or structural parameters μ and μ^*. It is immediate to verify that the same results would emerge in the presence of more than one domestic firm as long as entry of foreign firms takes place in equilibrium (in equilibrium we would still have two optimality conditions for domestic and foreign firms and a free entry condition binding on the foreign ones).[6] This proves our second result, which will be useful for the subsequent normative analysis:

THEOREM 2. **Under endogenous market structures with competition in quantities or in prices, any policy or structural shift directly affecting the profitability of domestic firms is not going to change the domestic consumers surplus and the strategies of the foreign firms, but changes their number and the profits of the domestic firms.**

Notice that a change in the ratio between size of the economy and fixed entry cost L/F has the same impact as before on the equilibrium system for the strategies of the foreign firms x^*, their aggregate statistic β^* and, as a consequence, for consumer surplus. Our model can be used to analyze the impact of trade in specific examples and the impact of structural changes between domestic and foreign firms or changes in the trade policy for different markets. The case of trade policy for a foreign market is analyzed in a companion paper (Etro, 2011a), while the cases of trade policy for a domestic market and for an integrated market are analyzed here.

First of all, let us interpret the relevant market as a domestic market of size L where the government can choose an import tariff $\mu^* = t$ and a production subsidy $\mu = \tau$. Assuming that the subsidy is a pure transfer of domestic resources and the tariff revenue from each foreign firm is a function $R(x^*, \beta^*)$,[7] welfare can be written as:

$$\begin{aligned} W(\tau, t) &= LU\left[(n-1)h(x^*) + h(x)\right] + \\ &\quad + L\Pi[x, (n-1)h(x^*), 0] - F + t(n-1)R(x^*, \beta^*) \end{aligned} \tag{7}$$

where the endogenous variables derive from the equilibrium characterized above and therefore from both policy tools, but the production subsidy does not affect

[6]What is important for our results below is that the number of domestic firms is given. In the short and medium run, it is reasonable to assume that entry and exit of international firms in a market can be quick (for instance because international firms are already active elsewhere). In the very long run, policy or structural shifts may also affect the number of domestic firms (for instance protectionism may induce the creation of new domestic business) up to the elimination of any residual domestic profits. Such a long run scenario has been already examined by Venables (1985) and Horstmann and Markusen (1986).

[7]Of course, under competition in quantities $R(x^*, \beta^*) = x^*$ and under competition in prices defined as above $R(x^*, \beta^*) = D(1/x^*, \beta^*)$.

consumer surplus in line with Theorem 2 and is a pure transfer of resources (it affects net domestic profits only indirectly). When available, the optimal subsidy must maximize the domestic profits net of the costs of the subsidy, since it does not affect consumer surplus. The optimal tariff must balance the benefits of profit shifting and tariff revenue with the costs of higher prices and less varieties. In Section 3 we will analyze the case of quantity competition with homogenous goods and show that, at least under constant marginal costs, a prohibitive subsidy is always optimal, but in its absence, an import tariff is optimal for profit shifting reasons only when the demand is not too convex. This confirms traditional results, but emerges for the first time in case of endogenous market structures.

As a second application, let us think of an integrated market of size L^W where the domestic firm is competing with international firms, and the domestic government can choose the subsidy $\mu = \tau$ while the international firms face an exogenous and common policy normalized at $\mu^* = 0$. The equilibrium must satisfy the same conditions as above, therefore the domestic subsidy does not affect the strategies of the international firms and consumer surplus, in line with Theorem 2. However, the subsidy reduces the number of firms and affects the strategy of the domestic firm. If the subsidy is a pure transfer of domestic resources, welfare can be written as:

$$W(\tau) = LU\left[\beta^* + h(x^*)\right] + L^W \Pi[x, (n-1)h(x^*), 0] - F \tag{8}$$

As usual, the optimal subsidy maximizes welfare taking into account the impact on the production or pricing strategies of the rivals. In traditional models with exogenous entry, this typically leads to a negative production subsidy under price competition and to a positive one under strategic substitutability (Eaton and Grossman, 1986, Section VI); moreover, a smaller relative size of the domestic market is going to increase the tax or reduce the subsidy. In the case of endogenous entry, however, the production subsidy for the integrated market does not affect the production or pricing strategies of the competitors, but only reduces their number, and, according to Theorem 2, it does not affect consumer surplus. Therefore, the optimal policy must take into account only the impact on foreign entry in the integrated market and its feedback on the domestic profits. As we will see in Section 4, this implies that it is always optimal to set a positive subsidy to domestic production, independent from the form of competition and from the relative size of the domestic market.

Other applications of our theorems concern the impact of other export promoting policies or of variations in transport costs, the equilibrium trade policy in the presence of lobbying activities, and the analysis of the strategies of multinationals with superior technologies that decide the form of foreign direct investment in international markets.[8]

2 Trade and competition in specific models

In this section we first discuss two classic applications of the general framework with endogenous market structures which will be used in the rest of the paper. The first is based on a model with homogenous goods and linear demand in the tradition of Brander (1981), Brander and Krugman (1983) and Horstmann and Markusen (1986), the second on the model with isoelastic demand in the tradition of Krugman (1980) but here analyzed under competition in quantities and in prices (rather than under monopolistic competition). In both cases we examine the impact of opening up to trade on competition and on welfare. Finally, we move to a general equilibrium set up that nests the neoclassical model, the monopolistic competition model and the Cournot model with homogenous goods.

2.1 The linear demand case

Let us start our examination of endogenous market structures from the only example which has been examined in the trade literature, the one of competition in quantities with linear demand functions. Consider an autarchic economy characterized by a market with L consumers with utility:

$$U = aC - \frac{C^2}{2} + Y \tag{9}$$

with C consumption of a good produced by n imperfectly competitive firms and Y numeraire good which is produced under perfect competition. Assuming a one-to-one transformation of labor in output in the competitive sector, the wage must be unitary and total income in the economy equals L. The demand of the first good is $C = a - p$ where p is the equilibrium price. Total production $X = L(a - p)$ can be inverted to derive the inverse demand $p = a - X/L$.

Under a linear cost function $c(x(i)) = cx(i)$, where c is the constant marginal cost (in labor units) and $x(i)$ the output of firm i, the profit function of this firm is:

$$\pi_i = \left(a - \frac{X}{L}\right) x(i) - cx(i) - F \tag{10}$$

[8]On the activity of multinationals in the presence of endogenous market structures Markusen and Stähler (2010) have independently obtained related results.

In Cournot equilibrium with endogenous entry the number of firms can be derived as $n = (a-c)\sqrt{L/F} - 1$, with individual production $x = \sqrt{LF}$ and price:

$$p = c + \sqrt{\frac{F}{L}} \tag{11}$$

Notice that welfare is $W = LU = X^2/2L$, which is clearly increasing in the total production and therefore decreasing in the equilibrium price.

Consider opening up to trade with an other country characterized by identical agents and total population L^*, with $L^W = L + L^*$ defined as the size of the integrated market. It is immediate to derive that the new equilibrium price is $p = c + \sqrt{F/L^W}$ which is reduced because of stronger competition in the integrated market: this is enough to conclude that welfare is increased with trade (in line with Theorem 1). The total number of firms $N = n + n^*$ becomes:

$$N = (a-c)\sqrt{\frac{L^W}{F}} - 2 \tag{12}$$

which is lower than the autarchic number $(a-c)(\sqrt{L/F} + \sqrt{L^*/F}) - 2$. Trade has increased concentration in the integrated market: this represents an example of beneficial concentration, because production becomes more efficient thanks to the reduction of spending in fixed costs. Consequently trade increases welfare (as in Brander and Krugman, 1983).

Similar results emerge under different technologies. For instance, with a quadratic cost function $c(x_i) = x(i)^2/2$, the equilibrium number of firms in the integrated market can be derived as:

$$N = a\sqrt{\frac{L^W}{F} + \frac{L^{W2}}{2F}} - 1 - L^W \tag{13}$$

which is again concave in the total labor force.

However, the equilibria characterized above are inefficient, typically generating excessive mark ups and excessive business creation (Mankiw and Whinston, 1986). This inefficiency is the reason for which the endogenous market structures approach gives rise to new implications for the impact of structural and policy shocks and to a new role for trade policy.

2.2 The isoelastic demand case

Let us move to the model with isoelastic preferences introduced by Dixit and Stiglitz (1977) and applied to trade theory by Krugman (1980). Consider a market with L agents with the following preferences for the differentiated goods:[9]

$$V = \left[\sum_{j=1}^{n} C(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{14}$$

where n is the number of goods and $\theta > 1$ is the elasticity of substitution. The consumer maximizes utility under the budget constraint $\sum_{j=1}^{n} p(j)C(j) = w$, where we normalize the individual labor supply to unity so that income is given by the wage w. Aggregate spending equals total income: $E = wL$. Each good i is produced according to the linear production function $x(i) = l(i)$, where $l(i)$ is the labor input used by firm i. Defining total demand of each good i as $x(i) = C(i)L$, the inverse and direct demands are:

$$p(i) = \frac{x(i)^{-\frac{1}{\theta}} E}{\sum_{j=1}^{n} x(j)^{\frac{\theta-1}{\theta}}} \quad \text{and} \quad x(i) = \frac{p(i)^{-\theta} E}{\sum_{j=1}^{n} p(j)^{1-\theta}} \tag{15}$$

Therefore, the profit function of each firm can be expressed in terms of production levels or prices as:

$$\pi_i = \frac{x(i)^{\frac{\theta-1}{\theta}} E}{\sum_{j=1}^{n} x(j)^{\frac{\theta-1}{\theta}}} - cx(i) - F = \frac{[p(i) - c]\, p(i)^{-\theta} E}{\sum_{j=1}^{n} p(j)^{-(\theta-1)}} - F \tag{16}$$

where c is the marginal cost of production: here we have $c = w$, to be normalized to unity so that $E = L$.

The Krugman model and most of the subsequent literature ignore the strategic interactions between firms, assuming monopolistic behavior. In equilibrium, each firm adopts the constant markup $\mu = \theta/(\theta-1)$ and the endogenous number of firms is $n = L/\theta F$.[10] Markups and production per firm are independent from the size of the economy L, while the number of firms is directly proportional to it. Consider opening up to trade with a country with L^* identical agents. The proportional increase in demand generates a larger number of varieties

[9] Total utility may take the form $U = U(V, Y)$ with Y numeraire and U quasilinear or homothetic (with total spending in the two sectors adjusted accordingly).

[10] The optimal price corresponds to a constant mark up on the unitary marginal cost $p = \theta c/(\theta - 1)$. Imposing the endogenous entry condition $(p - c)\, x = F$ for each firm, we have the equilibrium production of each good $x = F(\theta - 1)/c$. Normalizing $w = 1$ with $E = L$, the number of firms can be derived from the resource constraint equating the value of total sales and labor income $npx = L$, or from the market clearing condition for the labor market equating labor supply and labor demand, $L = n(F + x)$.

(Krugman, 1980):

$$N = \frac{L^W}{\theta F} \tag{17}$$

where L^W is the population of the integrated market. The price and the production of each firm remain the same as in the closed economy, but consumer surplus increase because more varieties can be consumed. In any symmetric equilibrium $V = (x/L)\, n^{\frac{\theta-1}{\theta}}$. However, the outcome is quite different when strategic interactions are taken in consideration: in the following subsections we analyze this case respectively under competition in quantities and in prices.

2.2.1 Competition in quantities

For the sake of simplicity, we start our investigation in the extreme case of homogenous goods ($\theta \to \infty$), in which the possibility of gains from variety is absent.

Consider first the equilibrium of a closed economy. Each firm has profits:

$$\pi_i = \frac{x(i)E}{X} - cx(i) - F \tag{18}$$

The output per firm in a symmetric Cournot equilibrium is $x = E(n-1)/n^2 c$, which implies a price $p = nc/(n-1)$ and generates profits $\pi = E/n^2 - F$. Now, let us normalize $c = w = 1$, which implies $E = L$ Under endogenous entry the number of producers is $n = \sqrt{L/F}$, which corresponds to a price $p = 1/\left(1 - \sqrt{F/L}\right)$ and to a production level $x = \sqrt{F}(\sqrt{L} - \sqrt{F})$. Contrary to what happens in the Krugman model, where price and production per firm are independent from the labor force, now the price is decreasing in the labor force and the production of each firm is increasing in it (to cover the fixed costs of entry with a smaller markup and a larger market share). This result is due to a competition effect associated with the positive impact of the size of the market on the equilibrium number of firms. However, the number of firms is increasing and concave in the size of the economy, exactly as in the model with a linear demand.

Consider opening up to trade with an other country characterized by identical agents so that the population of the integrated market is $L^W = L + L^*$. It is immediate to derive that the new equilibrium implies the price $p = 1/\left(1 - \sqrt{F/L^W}\right)$. The total number of firms becomes:

$$N = \sqrt{\frac{L^w}{F}} \tag{19}$$

which is lower than the number of firms active in autarchy, $\sqrt{L/F} + \sqrt{L^*/F}$. Nevertheless, the strengthening of competition in the integrated market leads to a lower international price level, and to a larger production for each firm. Trade has reduced the total number of firms, inducing an increase in world market concentration: in our case of homogenous goods, this represents an example of beneficial concentration, because production becames more efficient thanks to the reduction of the spending in fixed costs.

Of course, in case of imperfect substitutability between goods, product variety is beneficial and the impact of trade is more complex, but the competition effect of trade on prices and the reduction in the equilibrium number of varieties produced in each country persist. In particular, the first order condition for the maximization of profits (16) with respect to $x(i)$ imposing symmetry becomes:

$$\frac{(\theta-1)\,p}{\theta} - \frac{(\theta-1)\,p^2 x}{\theta E} = c \tag{20}$$

which can be solved for the Cournot equilibrium output in an integrated economy $x = (\theta-1)(N-1)E/\theta N^2 c$. Hence, the price is $p = \mu(\theta, N)c$ where the markup is:

$$\mu(\theta, N) = \frac{\theta N}{(\theta-1)(N-1)} \tag{21}$$

Using the normalization $w = 1$, profits in the integrated market become $\pi = (N+\theta-1)L^W/\theta N^2 - F$. Endogenous entry leads to the following number of firms under free trade:

$$N = \sqrt{\left(1-\frac{1}{\theta}\right)\frac{L^W}{\theta F} + \left(\frac{L^W}{2\theta F}\right)^2} + \frac{L^W}{2\theta F} \tag{22}$$

Opening up to trade (or an increase in the size of the integrated market) leads to a less than proportional increase of the number of firms and, consequently, to a reduction of the prices.[11] Notice that this equilibrium is inefficient compared to the monopolistic competition equilibrium characterized above because it generates higher mark ups and attracts an excessive number of firms (Mankiw and Whinston, 1986).

[11] Empirical research on endogenous market structures provides evidence on the competition effect outlined above (see Campbell and Hopenhayn, 2005 and, with specific reference to trade, Head and Ries, 1999, and Eaton, Kortum and Kramarz, 2004). Some novel empirical evidence in support of the less than proportional relation between market size and number of firms is provided in the working paper version of this article, which proposes an econometric test of the structural relation (17) derived from the Krugman model and of the alternative relations (19) or (22) derived from the EMS approach. Looking at panel data from different industries in the German manufacturing sector, evidence in support of the Cournot model with homogenous goods emerges quite clearly: estimating a structural relation as $\ln n = \beta_0 + \beta_1 \ln L + \varepsilon$, the coefficient β_1 appears significantly lower than one and close to 0.5.

2.2.2 Competition in prices

Under Bertrand competition, the first order condition for the maximization of profits (16) with respect to $p(i)$ imposing symmetry becomes:

$$x = (p - c)\left[\frac{\theta x}{p} - \frac{(\theta - 1)\, x^2}{E}\right] \tag{23}$$

In an integrated market, this can be solved for the Bertrand equilibrium price $p = \mu(\theta, N)c$ where the markup is given by:

$$\mu(\theta, N) = \frac{\theta(N-1)+1}{(\theta-1)(N-1)} \tag{24}$$

Using the normalization $w = 1$, profits are $\pi = L^W/(\theta N - \theta + 1) - F$. Endogenous entry leads to the price $p = \theta L^W/(\theta - 1)(L^W - F)$, which corresponds to the following number of goods/firms:

$$N = 1 + \frac{L^W - F}{\theta F} \tag{25}$$

The equilibrium production level is $x = F(\theta - 1)\left(L^W - F\right) / \left[L^W + (\theta - 1)F\right]$: in a larger market there are more firms and the strengthening of competition between them reduces the mark ups and increases the production of each firm. In this case, opening up to trade leads to a lower international price level without reducing the number of varieties produced in each country (or reducing it by at most one unit if we take in consideration the integer constraint on the number of firms), and increasing the production of each one. However, once again the beneficial impact of trade emerges from both a price reduction and an increase of the number of available varieties.

2.3 A 2x2x2 model

In this section we extend the model with isoelastic demand to a standard setup with two countries, two factors of production and two sectors. The inputs capital and labor are immobile and with endowments K and L for the home country and K^* and L^* for the foreign country. One sector is perfectly competitive and produces a homogenous good (agriculture), and the other is characterized by product differentiation and endogenous market structures with competition in prices or quantities and fixed entry costs (manufacturing).

Preferences are identical and homothetic in both countries over the homogeneous good Y and the differentiated goods, whose consumption delivers the consumption index V according to (14). Therefore, total utility is $U = V^\gamma Y^{1-\gamma}$, where $\gamma \in (0, 1]$ represents the relative preference for the bundle of differentiated goods. This implies that each agent spends a fraction γ of income in the differentiated goods.

Technology is also identical in both countries, with a CRS production function for the perfectly competitive sector:

$$Y = F(K_Y, L_Y) \tag{26}$$

where K_Y is capital employed in this sector and L_Y is the labor employed, which is associated with a constant marginal cost $c_Y(w, r)$. Increasing returns characterize the production of the differentiated goods. In particular, let us assume that the production of each variety requires a fixed cost of η units of labor and takes place according to a linearly homogenous function:

$$x = g(k_X, l_X) \tag{27}$$

where k_X and l_X are the inputs used by a representative firm. This function is associated with a constant marginal cost $c_X(w, r)$. Therefore, the profit function of each firm can be expressed as in (16) where total spending is a fraction γ of world income, $E = \gamma(wL + w^*L^* + rK + r^*K^*)$, the marginal cost is $c_X(w, r)$ and the fixed cost is $F = \eta w$. The same occurs in the foreign country, but the foreign wage w^* and rental rate r^* may be different, inducing different unit and fixed costs in both sectors. Free trade guarantees price equalization for the homogenous good, which is assumed to be the numeraire, and guarantees that the law of one price holds also for the differentiated goods. However, these goods may be produced in different quantities and may have different prices.

Let us assume that the capital-labor ratios of both countries are not too different and within a properly defined cone of diversification. Given the factor prices, by symmetry all domestic varieties are produced in quantity x and sold at the price p and all the foreign varieties are produced in quantity x^* and sold at price p^*. The equilibrium market structure is characterized by free entry conditions in the two sectors:

$$c_Y(w, r) = 1 \quad c_Y(w^*, r^*) = 1 \tag{28}$$

$$x[p - c_X(w, r)] = \eta w \quad x^*[p^* - c_X(w^*, r^*)] = \eta w^* \tag{29}$$

Under competition in quantities, the first order equilibrium conditions of the domestic and foreign firms - specular to (20) - can be rearranged as follows:

$$\frac{(\theta-1)p}{\theta} - \frac{(\theta-1)xp^2}{\theta E} = c_X(w, r) \quad \frac{(\theta-1)p^*}{\theta} - \frac{(\theta-1)x^*p^{*2}}{\theta E} = c_X(w^*, r^*) \tag{30}$$

For a given aggregate spending E, the six conditions (28)-(29)-(30) can be solved for the six unknowns w, w^*, r, r^*, x and x^* with prices p and p^* that can be derived from the inverse demand functions in (15).

Under competition in prices, the first order equilibrium conditions of the domestic and foreign firms - specular to (23) - are:

$$x = [p - c_X(w,r)]\left[\frac{\theta x}{p} + \frac{(\theta-1)x^2}{E}\right] \quad x^* = [p^* - c_X(w^*,r^*)]\left[\frac{\theta x^*}{p^*} + \frac{(\theta-1)x^{*2}}{E}\right] \tag{31}$$

The six conditions (28)-(29)-(31) can be solved for the six unknowns w, w^*, r, r^*, p and p^*, with sales of the domestic firms x and sales of the foreign firms x^* that can be derived from the direct demand functions in (15). The characterization of the equilibrium can be simplified extending a traditional result of neoclassical trade theory, factor price equalization, to the case of endogenous market structures:[12]

PRICE EQUALIZATION. **Under endogenous market structures in one sector and perfect competition in the other sector, a trading equilibrium with production of both countries in both sectors induces equal prices and output for all firms (under both price and quantity competition) and factor price equalization.**

This implies that all prices in manufacturing p are given by a common markup on the common marginal cost $c_X(w,r)$. The markup $\mu(\theta, N)$ changes with the two forms of competition but depends only on the elasticity of substitution θ and on the total number of firms $N = n+n^*$: it is still given by (21) under competition in quantities and (24) under competition in prices. The four market clearing conditions for the factor markets in both countries employ the Shepherd Lemma for which the labor (capital) requirement for a unitary production in sector $j = X, Y$ is the same in both countries and given by $a_{Lj} = \partial c(w,r)/\partial w$ ($a_{Kj} = \partial c(w,r)/\partial r$). Finally, the market clearing conditions for the integrated markets for goods can be combined in a single one by Walras' Law, so as to close the equilibrium. The system of eight equations:

$$c_Y(w,r) = 1 \tag{32}$$

[12] Helpman (1981) derived a similar result but with increasing returns at the industry level rather than at the firm level (that is, without a fixed cost but with decreasing average costs). Lahiri and Ono (1995) have obtained a similar result under Cournot competition with homogenous goods and under different technological conditions (see also Lawrence and Spiller, 1983, and Shimomura, 1998). Notice that the equalization of the prices of the imperfectly competitive firms is an assumption under homogenous goods, but is an additional result in our framework with differentiated goods.

$$x[p - c_X(w,r)] = \eta w \quad (33)$$

$$p = \mu(\theta, n + n^*)c_X(w,r) \quad (34)$$

$$a_{LY}Y + a_{LX}nx + \eta n = L \quad (35)$$

$$a_{LY}Y^* + a_{LX}n^*x + \eta n^* = L^* \quad (36)$$

$$a_{KY}Y + a_{KX}nx = K \quad (37)$$

$$a_{KY}Y^* + a_{KX}n^*x = K^* \quad (38)$$

$$(1-\gamma)(n+n^*)px = \gamma(Y + Y^*) \quad (39)$$

can be solved for the eight unknowns w, r, p, x, n, n^*, Y and Y^*. The amount of each input used in each sector of each country can be obtained residually, with $k_X = a_{KX}x$ and $l_X = a_{LX}x$. Let us assume that $a_{KX}/\left(a_{LX} + \eta/x\right) > a_{KY}/a_{LY}$ to make sure there are no factor intensity reversals in this framework. Solving the market clearing conditions of the inputs markets one obtains that when the population of the two countries is the same, the country with more capital produces more varieties of goods, and when the capital/labor ratio is the same, the country with a larger population produces more varieties of goods. Moreover, solving for nx/Y and n^*x/Y^*, we can immediately confirm another traditional result of neoclassical trade theory, the Heckscher-Ohlin theorem, in the presence of intraindustry trade and imperfect competition:

Pattern of trade. **Under endogenous market structures in a capital intensive sector and perfect competition in the labor intensive sector, a trading equilibrium with production of both countries in both sectors induces intraindustry trade in the capital intensive sector with positive net exports of the capital abundant country.**

Notice that, depending on the markup functions, the degree of substitutability between goods, and the size of the fixed costs relative to the size of the market, the model converges to traditional trade models:

- when the fixed cost shrinks (η is reduced) and substitutability between goods is increased (θ increases indefinitely) for a given population L^W, the model converges asymptotically to the Heckscher-Ohlin model with an increasing number of firms in the capital intensive sector and $\mu(\infty, \infty) \rightarrow 1$, so that both sectors are perfectly competitive in the limit: this suggests why the general model inherits some of the basic neoclassical properties on interindustry trade;

- when the population expands (L^W increases) for a given fixed cost η and a given degree of substitutability θ, the model converges to the Krugman model extended to general equilibrium as in Helpman and Krugman (1985), with monopolistic competition between an increasing number of firms in the capital intensive sector and $\mu(\theta, \infty) \to \theta/(\theta-1)$: this suggests why the general model inherits some of the basic properties of the monopolistic competition models on intraindustry trade;

- when substitutability between goods θ approaches infinity for a given fixed cost η and a given population L^W, the model converges to the Brander-Markusen approach with homogenous goods and competition in quantities as extended to free entry and general equilibrium by Lahiri and Ono (1995): this suggests why the general model inherits some of the basic properties of the imperfectly competitive framework in which trade increases competition.

Beyond these three particular cases, the general model presents simultaneously the three effects of trade: the impact of factor endowments on interindustry trade, the impact of product differentiation on intraindustry trade, and the impact of competition on markups. As a consequence, the general model also presents three sources of gains from trade: gains from comparative advantage related to factor endowments, gains from variety related to the increase in product differentiation and gains from competition related to the reduction of markups.

As an example, consider the case of Cobb-Douglas production functions:

$$Y = K^{\alpha_Y} L^{1-\alpha_Y} \quad \text{and} \quad x = k^{\alpha_X} l^{1-\alpha_X} \quad \text{with} \quad \alpha_X > \alpha_Y \tag{40}$$

which generate marginal cost functions $c_j(w,r) = w^{\alpha_j} r^{1-\alpha_j} / \alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}$ for $j = Y, X$. This allows us to solve explicitly for the entire equilibrium with closed form solutions depending on the form of competition.

To focus on the simplest case, let us assume competition in quantities with homogenous goods ($\theta \to \infty$), which implies $\mu = N/(N-1)$, and let us assume

$\gamma = 1/2$. The equilibrium number of firms can be derived as:[13]

$$N = \sqrt{\frac{L^W}{\eta} + \frac{\alpha_X^2}{4(2-\alpha_Y-\alpha_X)^2}} - \frac{\alpha_X}{2(2-\alpha_Y-\alpha_X)} \tag{41}$$

which allows to derive residually all the other equilibrium variables. The total number of firms is independent from capital endowments, but increases less than proportionally with the world endowment of labor (and this effect is made stronger by the introduction of capital, since the number of firms is decreasing in α_X and α_Y). This confirms that trade induces always a reduction of the markup and business destruction in at least one country. The allocation of firms between countries depends also on the capital endowments and, in particular, the number of domestic firms is increasing in the ratio between domestic and total capital.[14] The general case of imperfect substitutability (finite θ) with competition in quantities is more cumbersome but leads to analogous qualitative results.

In case of competition in prices with imperfect substitutability (finite θ) we can derive the general equilibrium number of firms in the integrated market as:

$$N = \frac{\frac{L^W}{\eta} + (\theta-1)(2-\alpha_Y-\alpha_X)}{[\theta(2-\alpha_Y-\alpha_X)+\alpha_X]} \tag{42}$$

which is now linearly increasing in world population, as in the Krugman model. However, trade exerts always a downward pression on the mark ups.

2.4 Implications

The analysis of endogenous market structures emphasizes three main effects of trade. First of all, trade integration strengthens competition and induces a

[13]From (32) and (34) we have $a_{LY} = (1-\alpha_Y)/w$ and $a_{LX} = (1-\alpha_X)p(N-1)/wN$, while from (33) and (34) we have $xp/N = \eta w$. Substituting these conditions and (39) into (35) and (36) and summing member by member, one can obtain an expression that can be solved for N as in the text. Replacing the previous conditions in (37) and (38) and summing member by member, one can obtain the number of domestic firms. Notice that in case of monopolistic competition with a constant markup $\mu = \theta/(\theta-1)$, one can also derive the total number of firms as $N = L^W/\eta\theta(1-\alpha_X-\alpha_Y)$. Closed form solutions are also available with technologies with fixed coefficients.

[14]In particular the number of domestic firms can be derived as:

$$n = (1-\alpha_Y)\frac{K}{K^W}N - \frac{\alpha_Y L}{\eta\left[\alpha_X(N-1)-\alpha_Y N\right]}$$

Notice that opening up to trade increases N and reduces the markup μ, but has ambiguous effects on the price because the marginal cost $c_X(w,r)$ is affected by general equilibrium changes of the factor prices. These have also an impact on income, which affects the welfare gains from trade.

generalized reduction of the markups, which is a primary source of gains from trade for consumers. As a second impact, the competition effect creates a negative feedback on profitability which crowds out the expansion of the number of produced varieties and leads to business destruction at the local level: this reduces the traditional gains from variety of the Krugman model with monopolistic competition. As a third impact, the equilibrium market structure leads to a larger production of each surviving firm: this increase in concentration is welfare enhancing because reduces the expenditure in fixed costs.

To clarify the quantitative implications of the endogenous market structure approach, let us consider the example of isoelastic demand with homogenous goods, and let us look at two identical countries opening up to trade. Suppose that ten firms are active in each country under autarchy. After the two countries open up to trade with each other, according to (19) or (41), only fourteen firms remain active - formally, if in the basic model under autarchy we have $n = n^* = \sqrt{L/F} = 10$, under free trade we must have $N = n\sqrt{2} \simeq 14$, with similar results in the 2x2x2 model. Moreover, if the autarchic markup is $\mu = 1/(1 - 1/10)$, the markup under free trade must be $\mu = 1/(1 - 1/14)$, which is equivalent to a reduction of the mark up from 11.1% to 7.7%. In partial equilibrium this leads to an equivalent price reduction. This is not necessarily the case in general equilibrium: part of the markup reduction may be crowded out by an in crease in the marginal cost due to changes in the factor prices (following the expansion of total factor endowments). Business destruction is indeterminate in partial equilibrium, but not in general equilibrium: for instance, in our case of identical countries each one of them looses three firms.

The relevance of the mechanisms associated with the strategic interactions depends on the type of traded goods under consideration. At one extreme we have perfectly differentiated goods with competition in prices: for these goods, most of the gains from opening up to trade derive from an increase in the number of consumed varieties, while business destruction is absent. At the other extreme we have homogenous goods with competition in quantities: for these, all the gains from trade derive from lower prices, but business destruction is heavy.

3 Import tariffs for domestic markets

In the theory of trade policy under imperfect competition, most of the investigations have been focused on the case of an exogenous number of domestic and foreign firms (active in a single factor economy) and on the incentives to adopt import tariffs for profit shifting reasons (Dixit, 1984; Cheng, 1988; Bhattacharjea, 1995).[15] In this setup, a positive import tariff is optimal as long as the demand function is not too convex and, when available, the optimal production subsidy is typically positive.

Endogenous entry has been considered only to study special cases and, in particular, the case of monopolistic competition in an integrated market (Flam and Helpman, 1987) and the case of Cournot competition, with domestic firms producing identical goods that are imperfectly substitutable for those produced by the foreign firms, in a segmented market (Venables, 1985) and in an integrated market (Horstmann and Markusen, 1986). Since in all these models both the domestic and foreign firms end up with zero profits (a reasonable assumption only in the very long run), any profit shifting rationale for protectionism disappears, and the optimal policy is only aimed at improving the terms of trade, exactly as in the neoclassical context. Therefore, the optimal tariff is always positive in the mentioned models with monopolistic competition (Helpman and Krugman, 1989) and with competition in quantities at least under linear demand - see the general analysis of Markusen and Venables (1988), but notice that, in the absence of transport costs and under perfect substitutability between domestic and foreign goods, any small tariff would drive completely out the foreign industry leaving domestic (zero) profits, consumer surplus and welfare unchanged compared to free trade.

A different situation emerges when we start from an equilibrium market structure where a domestic firm is competing with an endogenous number of international firms and we introduce policy tools that can affect entry of the foreign firms but not of the domestic ones. This is relevant when the international firms are already active elsewhere and their endogenous entry/exit in the domestic market is relatively quick, while the creation of new domestic firms requires much more time (or is excluded by domestic regulation).[16] In such a case, a profit shifting rationale for trade policy remains and the optimal policy balances the impact on consumer surplus through entry and pricing, on domestic profits and on tariff revenues. The purpose of this section is to analyze such an optimal policy for a segmented domestic market.

Under general product differentiation with different forms of competition, a

[15] Bhattacharjea (1995) takes free entry in consideration but without a commitment of the government on the tariff.

[16] To appreciate how realistic is our assumption notice that: 1) tariffs induce only the exit of foreign firms, which is certainly faster than entry; 2) our analysis goes through with minor changes in the presence of more than one domestic firm (as long as the number of domestic firms is fixed and residual endogenous entry of some international firms occurs in equilibrium); 3) free entry of domestic firms would drive out foreign firms and dissipate any domestic profits, therefore it may be in the same interest of the home country (part of the optimal trade policy) to limit domestic entry.

general derivation of the optimal trade policy is possible but complex, therefore we will mainly focus on the simpler case of homogenous goods. Notice that a tariff leads to an expansion of the domestic production, which increases domestic profits by reducing the average costs: this profit shifting effect is stronger when goods are highly substitutable. Moreover, the impact on consumer surplus is to increase prices and reduce the gains from variety due to the lower number of foreign firms active in the domestic market: the last effect is weaker when goods are highly substitutable. Therefore, the best case for using import tariffs emerges when goods are homogenous (and competition is in quantities).

Under homogenous goods, one can easily show that a positive production subsidy is always optimal and, when it is not available, the optimality of a positive tariff depends on the elasticity of the slope of the inverse demand $E \equiv Xp''(X)/p'(X)$. In the following proposition we restrict our attention to the case of constant marginal costs to prove a stronger claim:

Optimality of production subsidies and import tariffs. **Under endogenous market structures in the home market with competition in quantities, homogenous goods and constant marginal costs, the optimal subsidy to domestic production is prohibitive and, in its absence, the optimal import tariff is positive iff the elasticity of the slope of the inverse demand E is smaller than a positive cut-off, and is always negative if** $E > 2$.

The optimal import tariff under competition in quantities with homogenous goods, derived in the Appendix, is:

$$t_H = -x^* p'(X) \left[1 - \frac{1+v}{2} E \right] \tag{43}$$

where $\nu = x/X$ is the desired market share of the domestic firm. This formula differs from the optimal rule obtained by Dixit (1984) and Cheng (1988) for the case of a single domestic firm facing a fixed number of foreign firms. However, similarly to that case, a positive tariff requires the elasticity of the slope of demand to be small enough, here $E < 2/(1+\nu)$. Below we discuss the optimal policy in our usual two examples with homogenous goods: the linear case and the isoelastic case. In the former case $E = 0$, therefore we can derive explicitly the optimal positive tariff. In the latter case $E = 2$ and we will obtain an optimal import subsidy. Finally, we will briefly discuss the determinants of the optimal trade policy in case of product differentiation.

3.1 The linear demand case

Let us consider the model of the previous section with quasilinear preferences, where domestic income is spent in an imperfectly competitive sector characterized by competition in quantities or in a numeraire good produced under perfect competition. Initially, the first sector has an endogenous market structure with n firms, one of which is a domestic firm. The government chooses a specific tariff t on the imports from the foreign firms and possibly also a specific production subsidy τ on the sales of the national firm. With L domestic consumers, the inverse demand is $p = a - X/L$. The marginal cost of production is c and the fixed cost is F: this should be interpreted as a country-specific cost of production in the domestic country. The profit function of the domestic firm is $\pi = (a - X/L + \tau - c)x - F$, where x is its own production. The profit function of a foreign firm i is $\pi_i^* = (a - X/L - t - c)x^*(i) - F$ where $x^*(i)$ is its production. The Cournot equilibrium with endogenous entry can be easily derived. As long as there are foreign firms in the market and $t + \tau > 0$, the zero profit condition must be binding on them, which implies $n = (a - c - 2t - \tau)\sqrt{L/F} - 1$, with a production of the foreign firms $x^* = \sqrt{LF}$, which is independent from the subsidy in line with Theorem 2. Total production is:

$$X = (a - c - t)\,L - \sqrt{LF} \tag{44}$$

and the production of the domestic firm can be derived as:

$$x = \sqrt{LF} + (\tau + t)\,L$$

Such an equilibrium is consistent with $n \geq 2$ for $t \leq (a - c - \tau - 3\sqrt{F/L})/2$. If this is the case, welfare is the sum of consumer surplus, domestic profits and tariff revenues net of the subsidy cost:

$$W = \frac{X^2}{2L} + \left(\frac{x^2}{L} - F\right) + t(X - x) - \tau x \tag{45}$$

where the subsidy does not affect consumer surplus, in line with Theorem 2, but the import tariff affects all the components of welfare. Welfare turns out to be a linearly increasing function of the subsidy: therefore, if both the instruments are available, the first best can be obtained by setting $\tau_H = a - c$ and $t_H = 0$ which excludes entry of foreign firms.

Consider now the case in which the subsidy is not available. Maximizing welfare, the optimal unilateral tariff can be derived as:[17]

$$t_H = \sqrt{\frac{F}{L}} \tag{46}$$

[17] Of course, the optimal tariff can be derived from the general rule (43) setting $E = 0$ since $x^* = \sqrt{LF}$ and $p'(X) = -1/L$.

which is consistent with entry of some foreign firms as long as the fixed cost is small enough (otherwise, it is optimal to set the prohibitive tariff). Contrary to what happens with a given number of firms, the optimal tariff does not depend on $a-c$, but it increases in the ratio F/L; moreover, the tariff does not affect the production of the foreign firms, which remains the same. The main role of the tariff is to reduce entry of these firms, which reduces the production inefficiency associated with the free trade equilibrium - due to the excessive prices and entry *à la* Mankiw and Whinston (1986). This creates space for increasing the market share and the profits of the domestic firm (compared to the zero profits under free trade), while collecting also some tariff revenue. These benefits are larger when the fixed cost is high relative to the size of the market because in this case the production inefficiency associated with free trade is high. In the limit case of perfect competition (zero fixed costs) we obtain the traditional result for which free trade is optimal.

The equilibrium production of the domestic firm is $x = 2\sqrt{LF}$, which is twice as the production of the other firms, and the domestic profits are $\pi = 3F$. Total imports are $IMP = (a-c)L - 4\sqrt{LF}$, which delivers a tariff revenue $(a-c)\sqrt{LF} - 4F$. It is important to remark that the same outcome could be obtained setting an import quota $Q = IMP$ and auctioning the rights to sell at a price t_H: endogenous entry of foreign producers would deliver the same allocation of production as above.[18]

While the domestic economy gains from the positive profits of the domestic firm and from the revenue collected from the foreign firms, its consumers have to face a higher equilibrium price $p = c+2\sqrt{F/L}$ compared to the one emerging under free trade, given by (11).When the size of the market becomes large, the optimal tariff tends to zero, a result that is consistent with the traditional idea that free trade is optimal when the domestic firms (here only one) are too small to affect the terms of trade.

A last comment is due on the case in which a long run commitment to an import tariff is possible and endogenous entry of domestic firms can take place as a consequence. Such a case has been analyzed by Venables (1985) and Markusen and Venables (1988), under the assumption that all domestic firms produce the same good and all foreign firms produce the same or a different good, showing that domestic entry dissipates any profit shifting rationale for trade policy, whose only aim can be to affect the terms of trade. However, in case of homogenous goods (without transport costs) any tariff or subsidy eliminates the entire production in one of the two countries (with specialization

[18] A smaller quota would ask a higher bidding price for the rights to sell, and would deliver a different equilibrium: the domestic firm would also reduce its production. However, it is immediate to verify that the optimal quota reproduces the equilibrium with the optimal tariff.

in the residual perfectly competitive sector), without affecting welfare relative to free trade. In contrast, we have shown that a country can always gain from a production subsidy or an import tariff as long as domestic entry in the market is limited: protectionism and entry regulation are complementary unilateral policies.

3.2 The isoelastic demand case

Let us consider the simplest case emerging from a quasilinear utility $U = \log C + Y$, where both consumption goods are homogenous, but produced under perfect competition (Y) and under competition in quantities (C). Clearly this corresponds to our usual isoelastic case with a consumption index (14) characterized by $\theta \to \infty$. With L domestic consumers, the inverse demand is $p = L/X$. Notice that the elasticity of the slope of the inverse demand is $E = 2$, therefore we expect welfare to be decreasing with the introduction of a small tariff.

The profit function of the domestic firm is $\pi = (L/X - c + \tau)\, x - F$, and the profit function of a foreign firm i is $\pi_i^* = (L/X - c - t)\, x^*(i) - F$. In the Cournot equilibrium with endogenous entry total production can be derived as:

$$X = \frac{\sqrt{L}\left(\sqrt{L} - \sqrt{F}\right)}{c + t} \tag{47}$$

with a production of the foreign firms $x^* = (\sqrt{LF} - F)/(c+t)$ and a production of the domestic firm given by:

$$x = \frac{\left(c\sqrt{F} + t\sqrt{L}\right)\left(\sqrt{L} - \sqrt{F}\right)}{(c+t)^2} + \frac{\tau}{(c+t)\sqrt{L}}$$

The tariff reduces total and foreign production and increases domestic production, while the subsidy only increases the latter, in line with Theorem 2. Welfare becomes:

$$W = L \log\left(\frac{X}{L}\right) + \left(\frac{L}{X} - c\right) x - F + t\,(X - x) \tag{48}$$

where again the subsidy does not affect consumer surplus, but the import tariff affects all the components of welfare. One can verify that the optimal subsidy is prohibitive also in this case. However, when such a tool is not available, a small tariff is welfare reducing. Maximizing welfare, one can actually derive an optimal import subsidy:

$$t_H = -\frac{F}{L} \tag{49}$$

Notice that also in this case, the intervention is positively related to the size of the fixed costs: when the price distortion due to the limited entry is large, it is convenient to subsidize entry and reduce the prices even if this reduces domestic profits. However, when the size of the domestic market is large the subsidy is reduced and free entry is optimal (as we expect for a small open economy with a domestic firm which is negligible compared to the other firms).

In case of product differentiation, namely reducing the elasticity of substitution θ between goods, the optimal trade policy should take in consideration the gains from variety in the domestic market. This implies that a prohibitive production subsidy is not anymore optimal when differentiation is high enough (θ is low). In the absence of production subsidies, the optimal tariff balances the usual effects plus the gains from varieties: since an import subsidy is optimal under homogenous goods, it is likely to remain optimal under imperfect substitutability. Analogous considerations apply under product differentiation with competition in prices.

4 Production subsidies for integrated markets

The new trade theory associates the scope of strategic trade policy in the presence of international oligopolies with profit shifting, showing that the optimal unilateral policy is typically a tax on exportable domestic production under price competition (Eaton and Grossman, 1986) and a subsidy to domestic production under quantity competition (Dixit, 1984).[19] More precisely, when part of the production is consumed domestically in an integrated market (Eaton and Grossman, 1986, Section VI), the optimal policy must take in consideration also the impact on the domestic consumer surplus: typically, this tends to bias the optimal policy toward lower taxes or higher subsidies to reduce the prices, and such a bias is stronger when the size of the domestic country is large relative to the rest of the integrated market. However, the nature of the optimal unilateral policy for an integrated market remains ambiguously dependent on the form of competition.

In this section we analyze the optimal unilateral policy for an integrated market which is initially characterized by an endogenous structure and, for simplicity, by a single domestic firm. Coherently with the previous section, the analysis is confined to the case in which there is always endogenous entry/exit of international firms but the domestic firm remains the only target of the domestic

[19] When export subsidies are forebidden, as under the WTO rules, countries can rely on subsidies to the entire domestic production. These are our focus here (see Etro, 2011a, on the role of pure export subsidies, R&D subsidies and competitive devaluations).

trade policy. As we will see, this gives raise to a new case for adopting production subsidies for profit shifting reasons. Our situation can be seen as the relevant one (in the short/medium run) when the subsidies are firm-specific or the number of domestic firms remains fixed by supply or regulatory constraints. The case of endogenous entry of both domestic and foreign firms has been already analyzed by Horstmann and Markusen (1986): in that case any profit shifting rationale for intervention disappears and the only rationale for trade policy (once again for a tax on domestic production) relies on the impact on the terms of trade in general equilibrium, just like in the neoclassical theory.[20]

Contrary to traditional analysis with a fixed number of international firms in the integrated market, we can show that the optimal unilateral policy is unambiguously a positive subsidy to domestic production. We first state a general result that follows from Theorem 2 and then discuss its applications:

Optimality of domestic production subsidies. **Under endogenous entry in the integrated market, the optimal subsidy to domestic production is always positive and independent from the relative size of the domestic market; with competition in quantities, homogenous goods and constant marginal costs, the optimal subsidy is prohibitive.**

The logic of this result is simple: since a production subsidy for an integrated market is not going to affect domestic consumer surplus by Theorem 2, it should be used to maximize domestic profits net of the cost of the subsidy. This always requires a positive subsidy finalized to reduce the price of the domestic firm, expand domestic production and limit the number of foreign competitors. In the Appendix we also derive the optimal subsidy, which can be prohibitive if goods are substitute enough and the marginal cost is not too much increasing. Under competition in quantities with inverse demand $p(x(i), \beta_i)$ with $\beta_i = \sum_{j \neq i} h(x(j))$, the optimal non-prohibitive subsidy is:

$$\tau_H = -p_2\left(x, \beta\right) h'(x)x \tag{50}$$

which, under homogenous goods, is inversely related to the elasticity of demand. Under competition in prices with direct demand $D(p(i), \beta_i)$ where $\beta_i =$

[20] Horstmann and Markusen (1986) and Markusen and Venables (1986) have shown that a tax on exportable domestic production is optimal. However, their model is highly specific, since it is based on competition in quantities with linear demand and perfect substitutability between the goods produced in each country. Moreover, when the goods produced in one country are also perfectly susbstitutable with the goods produced in the other country (the only case directly comparable to our model), any small tax or subsidy eliminates production in one country (with specialization in the residual perfectly competitive sector) without affecting welfare. In contrast, we show that a country can always gain from production subsidies as long as domestic entry in the integrated market is limited.

$\sum_{j\neq i} h(1/p(j))$, the optimal non-prohibitive production subsidy is:

$$\tau_H = \frac{(p-c)\Delta\,(p,\beta)}{|D_1\,(p,\beta)|} \tag{51}$$

where $\Delta\,(p,\beta) > 0$ is the positive indirect effect that a price increase exerts on demand through the change in the endogenous number of firms. The stronger is this effect (for instance because goods are close substitutes), the larger should be the optimal subsidy. Moreover, the optimal susbidy does not depend on the relative size of the domestic economy. It is the same whether the domestic country exports its entire production or only part of it. The reason is that profits are gained worldwide, while the consumer surplus is not affected by the subsidy. Below we provide few examples in our usual linear and isoelastic applications.

4.1 The linear demand case

Let us first consider the optimal unilateral policy for an integrated market in case of homogenous goods and linear demand. Consider the usual inverse demand $p = a - X/L^W$ where the size of the integrated market is L^W.

In case of a constant marginal cost c we could easily characterize the endogenous market structure as depending on the production subsidy and verify that welfare is always increasing in the subsidy. Therefore, the optimal trade policy requires the prohibitive subsidy:

$$\tau_H = a - c - 3\sqrt{F/L^W} \tag{52}$$

In the case of a quadratic cost function $c(x) = x^2/2$, a prohibitive subsidy is typically suboptimal because it induces inefficient production. Looking at the Cournot equilibrium between N firms for a given subsidy τ of the domestic firm, and imposing the endogenous entry condition, we can obtain the same equilibrium production for each international firm as under free trade, $x^* = \sqrt{2L^W F/(2+L^W)}$, and the number of firms:

$$N = \left(a - \frac{\tau}{1+L^W}\right)\sqrt{\frac{(2+L^W)L^W}{2F}} - 1 - L^W$$

which imply total production $X = aL^W - (1+L^W)x^*$ and a price $p = (1+L^W)\sqrt{2F/\left(2+L^W\right)L^W}$. The equilibrium production of the subsidized firm is:

$$x(\tau) = x^* + \frac{\tau L^W}{1+L^W}$$

Consistently with Theorem 2, the subsidy does not affect the individual production of the other firms, but decreases their number. Domestic welfare is:

$$W = \frac{1}{2}\left(\frac{X}{L^W}\right)^2 L + \left(x(\tau)p - \frac{x(\tau)^2}{2} - F\right) \tag{53}$$

where the first term is independent from τ. Welfare is maximized when:

$$x(\tau) = p = \left(1 + L^W\right)\sqrt{\frac{2F}{\left(2 + L^W\right)L^W}}$$

which requires the following optimal subsidy:

$$\tau_H = \left(1 + \frac{1}{L^W}\right)\sqrt{\frac{2F}{\left(2 + L^W\right)L^W}} > 0 \tag{54}$$

This is approaching zero when the size of the market increases indefinitely. Notice also that when the fixed cost of entry decreases, the level of concentration in the market is reduced and the optimal subsidy goes down: in the limit case of perfect competition (zero fixed costs) we obtain the traditional result for which free trade is optimal.

4.2 The isoelastic demand case

Let us consider the usual isoelastic preferences (14). Consider competition in quantities first. The inverse demand (15) can be rewritten here as:

$$p(x, \beta) = \frac{x^{-1/\theta}L^W}{x^{1-1/\theta} + \beta}$$

with $h(x) = x^{(\theta-1)/\theta}$. Applying directly the formula for the optimal production subsidy (50) we obtain $\tau_H = (\theta - 1)p^2 x/\theta L^W$, which leads the domestic firm to sell its good at a price $p = c\theta/(\theta - 1)$ lower than the other firms. This allows us to rewrite the optimal export subsidy as:

$$\tau_H = \frac{p}{(L^W/cx)} \tag{55}$$

which is decreasing in the size of the international market L^W. In other words, large countries exporting in small markets (relative to the domestic production) should adopt large subsidies for their exporting firms, while small open economies (exporting to large markets) should tend to commit to free trade.

Let us move to the case of price competition. The direct demand function derived from our usual CES preferences in (15) can be rewritten here as:

$$D(p,\beta) = \frac{p^{-\theta}L^W}{p^{1-\theta}+\beta}$$

with $h(1/p) = p^{1-\theta}$. Under the optimal policy, the equilibrium price of the domestic firm must be $p = \theta c/(\theta-1)$, exactly as in the case of competition in quantities. Moreover, the foreign firms must sell at the same price of free trade, $p^* = \theta c L^W/(\theta-1)\left(L^W - F\right)$, which is higher than p. Applying (51), the associated optimal subsidy to domestic production can be derived as:

$$\tau_H = \frac{p}{\theta\left\{(1-F/L^W)^{\frac{1}{\theta}}\left[L^W/\theta F + (\theta-1)/\theta\right] - 1\right\}} > 0 \qquad (56)$$

which is again decreasing in the size of the international market.

5 Conclusion

We studied international trade theory in the presence of endogenous market structures characterized by both strategic interactions and endogenous entry of firms. The theoretical analysis has emphasized that globalization leads to lower prices and to fewer, but larger, firms in a country. This increases welfare but leads to an inefficient organization of the global production. For this reason we have revisited the role of trade policy in the presence of endogenous entry in domestic and integrated markets. Moreover, we have developed a 2x2x2 general equilibrium models that nests the neoclassical framework, the Krugman model and the approach based on imperfect competition. The specific and general models analyzed here could be used for new investigations of traditional trade topics, as the role of multinationals, the implications of international factor movements, the impact of tariffs in the general equilibrium 2x2x2 model, the effects of other forms of protectionism, the outcome of lobbying activity for trade policy and more.

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Appendix

Price equalization in the 2x2x2 model. Consider competition in quantities first. From the inverse demand (15) we can obtain the price of a domestic variety as $p = E/x^{1/\theta}\Sigma$ and the price of a foreign variety as $p^* = E/x^{*1/\theta}\Sigma$ where $\Sigma = nx^{\frac{\theta-1}{\theta}} + n^*x^{*\frac{\theta-1}{\theta}}$ is the usual aggregate statistic of the quantity strategies. With this, we can rewrite the market equilibrium equations as:

$$\begin{array}{cc} c_Y(w,r)=1 & c_Y(w^*,r^*)=1 \\ \frac{Ex^{\frac{\theta-1}{\theta}}}{\Sigma} - c_X(w,r)x = \eta w & \frac{Ex^{*\frac{\theta-1}{\theta}}}{\Sigma} - c_X(w^*,r^*)x^* = \eta w^* \\ \frac{(\theta-1)}{\theta\Sigma^2}Ex^{-\frac{1}{\theta}}\left(\Sigma - x^{\frac{\theta-1}{\theta}}\right) = c_X(w,r) & \frac{(\theta-1)}{\theta\Sigma^2}Ex^{*-\frac{1}{\theta}}\left(\Sigma - x^{*\frac{\theta-1}{\theta}}\right) = c_X(w^*,r^*) \end{array}$$

Given world expenditure E and given an aggregate statistic $\Sigma = nh(x)+n^*h(x^*)$ with $h(x) = x^{(\theta-1)/\theta}$, we have two specular systems of three equations respectively in (w,r,x) and in (w^*,r^*,x^*). If the mapping of the two systems is univalent, then both countries will have the same factor prices, the same level of output and the same price of a firm in the imperfectly competitive sector.

Consider competition in prices now. From the inverse demand (15) we can obtain the direct demand of a domestic variety $x = Ep^{-\theta}/P$ and the direct demand of a foreign firm $x^* = Ep^{*-\theta}/P$, where $P = np^{1-\theta} + n^*p^{*1-\theta}$ is a standard price statistic. With this, we can rewrite the equilibrium equations as:

$$\begin{array}{cc} c_Y(w,r)=1 & c_Y(w^*,r^*)=1 \\ \frac{Ep^{-\theta}[p-c_X(w,r)]}{P} = \eta w & \frac{Ep^{*-\theta}[p^*-c_X(w^*,r^*)]}{P} = \eta w^* \\ 1 = [p - c_X(w,r)]\left[\frac{\theta}{p} + \frac{(\theta-1)p^{-\theta}}{P}\right] & 1 = [p^* - c_X(w^*,r^*)]\left[\frac{\theta}{p^*} + \frac{(\theta-1)p^{*-\theta}}{P}\right] \end{array}$$

Given world expenditure E and given an aggregate statistic $P = nh(1/p) + n^*h(1/p^*)$ with $h(x) = x^{\theta-1}$ with $x = 1/p$, we have two specular systems of three equations respectively in (w,r,p) and in (w^*,r^*,p^*). If the mapping of the two systems is univalent, then both countries will have the same factor prices, the same price and the same output of a firm in the imperfectly competitive sector. □

Pattern of trade in the 2x2x2 model. Exploiting the common production of all firms x, one can solve the equilibrium conditions of the factor markets for n, n^*, Y and Y^* to obtain:

$$n = \frac{\left(\frac{K}{L} - \frac{a_{KY}}{a_{LY}}\right) L}{\frac{a_{KX}}{a_{LX}+\eta/x} - \frac{a_{KY}}{a_{LY}}} \qquad n = \frac{\left[K^*/L^* - a_{KY}/a_{LY}\right] L^*}{\left[\frac{a_{KX}}{a_{LX}+\eta/x} - \frac{a_{KY}}{a_{LY}}\right]}$$

It follows that $n \gtrless n^*$ if and only if $K \gtrless K^* + (a_{KY}/a_{LY})(L - L^*)$. At the same time we can solve for Y and Y^* and derive the relative production in the two sectors:

$$\frac{nx}{Y} = \frac{\left(\frac{K}{L} - \frac{a_{KY}}{a_{LY}}\right) a_{LY}}{\left(\frac{K}{L} - \frac{a_{KX}}{a_{LX}+\eta/x}\right)(a_{LX} + \eta/x)} \qquad \frac{n^*x}{Y^*} = \frac{\left(\frac{K^*}{L^*} - \frac{a_{KY}}{a_{LY}}\right) a_{LY}}{\left(\frac{K^*}{L^*} - \frac{a_{KX}}{a_{LX}+\eta/x}\right)(a_{LX} + \eta/x)}$$

Under the assumption that $\frac{a_{KX}}{a_{LX}+\eta/x} > \frac{a_{KY}}{a_{LY}}$, we immediately obtain that:

$$\frac{nx}{Y} \gtrless \frac{n^*x}{Y^*} \text{ iff } \frac{K}{L} \gtrless \frac{K^*}{L^*}$$

Since both countries consume goods in the same proportions because of the homothetic preferences, and in particular the market clearing condition for the goods markets provide $(n + n^*)x/(Y + Y^*) = \gamma/(1 - \gamma)p$, we must have that the country which is relatively abundant of capital produces relatively more differentiated good than it consumes, and therefore exports them. □

Optimality of production subsidies and import tariffs. Assume competition in quantities, homogenous goods with inverse demand $p = p(X)$ and constant marginal costs c. The endogenous market structure is characterized by the following optimality and free entry conditions:

$$\begin{aligned} p(X) + xp'(X) &= c - \tau \\ p(X) + x^*p'(X) &= c + t \\ \left[p(X) - c - t\right] x^* &= F \end{aligned}$$

under the second order condition $E < 2X/x$. Welfare can be written as:

$$W = \int_0^X p(s)ds - p(X)X + [p(X) - c]x - F + t[X - x]$$

whose differentiation with respect to τ provides:

$$\frac{\partial W}{\partial \tau} = [p(X) - c - t]\frac{dx}{d\tau} = \frac{F}{x^*(-p'(X))}$$

since $dx = -d\tau/p'(X)$ from the equilibrium system. Since welfare is always increasing in the subsidy as long as there is entry of foreign firms, whenever the subsidy is available the optimal trade policy is prohibitive.

Differentiation of welfare with respect to t gives:

$$\frac{\partial W}{\partial t} = [p(X) - c - t]\frac{dx}{dt} - [p'(X)(X - x) - t]\,\frac{dX}{dt} + X - x$$

The equilibrium system provides:

$$\frac{dX}{dt} = \frac{2(p(X) - c - t)}{2(p(X) - c - t)p'(X) + Fp''(X)} = \frac{2}{2p'(X) + x^{*}p''(X)} < 0$$

where we used repeatedly the equilibrium conditions and the sign derives from the second order condition of the foreign firms, and:

$$\frac{dx}{dt} = \frac{dx}{dX}\frac{dX}{dt} = \left(\frac{p' + xp''}{-p'}\right)\frac{dX}{dt}$$

Replacing these conditions, the welfare maximizing tariff can be obtained as:

$$t_H = -x^{*}\left[p'(X) + xp''(X)\right] - \frac{(X - x)\,x^{*}p''(X)}{2}$$

which can be rearranged as in the text.

To verify when a positive tariff is optimal, let us evaluate the optimality condition for the import tariff at free trade:

$$\begin{aligned} \frac{\partial W}{\partial t}\lfloor_{t,\tau=0} &= [p(X) - c]\frac{dx}{dt}\lfloor_{t,\tau=0} + \frac{(n-1)X}{n}\left[1 - p'(X)\frac{dX}{dt}\lfloor_{t,\tau=0}\right] = \\ &= \frac{(p - c)^2\left[E(1+n) - 2n\right]}{\left[2(p - c)p'(X) - Fp''(X)\right]n} \gtrless 0 \;\text{ iff }\; E \lessgtr \bar{E} \equiv \frac{2n}{n+1} \end{aligned}$$

where we used the comparative statics results at free trade. Notice that $n \to \infty$ implies $\bar{E} \to 2$ and $n = 2$ implies $\bar{E} = 4/3$. Since $\bar{E}$ depends on the number of firms in free trade, it depends on exogenous parameters as the fixed cost and the size of the market and we can conclude that for a given $\bar{E} \in [4/3, 2)$, the condition $E < \bar{E}$ is a sufficient condition for a small tariff to be welfare improving and $E \geq 2$ is a sufficient condition for a small tariff to be welfare reducing. □

Optimality of domestic production subsidies. Consider first a general model of competition in quantities with a specific subsidy. Assuming a general inverse demand function and a general cost function, the profit function of firm i facing a specific subsidy τ_i is:

$$\pi_i = x(i)[p(x(i), \beta_i) + \tau_i] - c(x(i)) - F$$

where $c(x)$ is a positive and increasing cost function. Let us focus on the case of a subsidy τ for the domestic firm and no subsidy for the other firms. The optimality and endogenous entry conditions in equilibrium are:

$$\begin{aligned} p(x^*, \beta^*) + x^* p_1(x^*, \beta^*) &= c'(x^*) \\ p(x, \beta) + x p_1(x, \beta) &= c'(x) - \tau \\ x^* p(x^*, \beta^*) - c(x^*) &= F \end{aligned}$$

where $\beta^* = (n-2)h(x^*) + h(x)$ is the spillover received by an international firm from the strategies of all the other firms in the market and $\beta = (n-1)h(x^*)$ is the spillover for the domestic firm. We know from Theorem 2 that the production of the foreign firms x^* and their spillovers β^* do not depend on the subsidy, while $x(\tau)$ and $\beta(\tau)$ depend on it. Therefore, we can write the domestic welfare as:

$$W = Lu\left[\beta^* + h(x^*)\right] + x(\tau)\left[p\left(x(\tau), \beta(\tau)\right) - c(x(\tau))\right] - F$$

where the first term is policy-independent by Theorem 2. Welfare maximization entails an interior solution for the optimal subsidy if goods are poor substitutes or if marginal costs are increasing enough: if this is not the case, a prohibitive subsidy is optimal. In particular, under homogenous goods, that is with inverse demand $p = p(X)$, we have:

$$dW = \left[p(X) - c'(x(\tau))\right] x'(\tau) d\tau$$

which is always positive for $c'(x)$ constant, decreasing or weakly increasing: in such a case, a prohibitive subsidy is optimal. Consider the case of an interior solution. Deriving the welfare maximizing production subsidy (and using the first order condition for the domestic firm) we obtain (50) in the text. Notice that the optimal production subsidy is always positive and does not depend on the relative size of the domestic country: it is the same whether the firm is exporting its entire production or only a part of it.

Consider now a general model of price competition with a specific subsidy τ_i for firm i, such that its profits are given by:

$$\pi_i = \left(p(i) - c + \tau_i\right) D\left(p(i), \beta_i\right) - F$$

where c is the constant marginal cost and the demand function is decreasing in both arguments with $\beta_i = \sum_{j \neq i} h(1/p(j))$ for positive: this implies that the demand for the domestic good is decreasing in its price and increasing in all the other prices. Substitutability between goods is guaranteed by the fact that the cross derivative $\partial D_i / \partial p(j) \equiv \Delta_{ij}$ is always positive: $\Delta_{ij} = -D_2 h' / p(j)^2 > 0$

for any i and j. The equilibrium conditions are:

$$\begin{aligned}(p^* - c)D_1(p^*, \beta) + D(p^*, \beta) &= 0 \\ (p - c + \tau)D_1(p, \beta) + D(p, \beta) &= 0 \\ (p^* - c)D(p^*, \beta^*) &= F\end{aligned}$$

where $\beta^* = (n-2)h(1/p^*) + h(1/p)$ is the spillover received by an international firm from the strategies of all the other firms in the market and $\beta = (n-1)h(1/p^*)$ is the spillover for the domestic firm. Theorem 2 tells us that the price of the foreign firms p^* and their spillover β^* do not change with the subsidy, while the price of the domestic firm $p(\tau)$ and its spillover $\beta(\tau)$ depend on the subsidy. Moreover, since consumer surplus depends on the usual aggregate statistic, in equilibrium $U[\beta^* + g(p^*)]$ is independent from the subsidy. Therefore, we can write total welfare of the domestic country as:

$$W = Lu\left[\beta^* + h(1/p^*)\right] + \left[p(\tau) - c\right] D\left[p(\tau), \beta(\tau)\right] - F$$

where the first term is policy-independent. Let us define $\Delta(p, \beta) = -D_2(p, \beta)\, h'(1/p)/p^2$ as the indirect effect that a price change exerts on demand through the change in the endogenous number of firms. Maximizing welfare one obtains the optimal subsidy to domestic production (51), which is always positive and depends on the impact of a price change on the entry of competitors ($\Delta(p, \beta)$). $\square$