Innovation, Firm Efficiency, and Market Structure

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Abstract: While numerous empirical studies have investigated whether efficiency is behind firm growth and industry market structure, there are few theoretical models underlying this work. We develop a theoretical model of endogenous firm growth that conforms to empirical regularities found in the literature and explicitly defines firm efficiency as the principal driver of firm growth and market structure. Estimating a stochastic frontier model with U.S. manufacturing firm data from the COMPUSTAT database, we find results consistent with the theoretical model. This model serves as a unified framework for additional theoretical and empirical work that explicitly incorporates both innovation and firm efficiency.

JEL Classifications: L11 & O31

Key words: Firm Efficiency, Demsetz Hypothesis, Firm R&D and Innovation, Firm Growth and Market Structure

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Abstract: While numerous empirical studies have investigated whether efficiency is behind firm growth and industry market structure, there are few theoretical models underlying this work. We develop a theoretical model of endogenous firm growth that conforms to empirical regularities found in the literature and explicitly defines firm efficiency as the principal driver of firm growth and market structure. Estimating a stochastic frontier model with U.S. manufacturing firm data from the COMPUSTAT database, we find results consistent with the theoretical model. This model serves as a unified framework for additional theoretical and empirical work that explicitly incorporates both innovation and firm efficiency.

I. Introduction

Of long standing interest has been the relationship between a firm's innovative efforts, efficiency and market structure. Research on these themes have typically either evaluated Schumpeter's (1942) notion of market power encouraging firm innovation or Demsetz's (1973) hypothesis that dominant firms owe efficiency and not market power for their position. While numerous, and often controversial, empirical studies[1] have produced mixed results in these areas, there are few theoretical models that account for the patterns observed in the data. Where models of R&D and innovation are found they are "typically based on econometric models without much theoretical content….[or] focusing on macro issues and a few stylized facts (Klette & Griliches, 2000)." Following recent efforts that borrow from macroeconomic theories of endogenous growth, we develop an endogenous model of firm innovation that: (i) generates larger market shares for efficient firms; and (ii) drives firm growth through static and dynamic inefficiency produced, and not imposed, by the model. This model can serve as unified framework

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[1] Cohen & Levin (1989), Bresnahan (1989), & Schmalensee (1989) offer thorough reviews of the empirical literature.

for additional theoretical and empirical work that explicitly incorporates both innovation and firm efficiency into models of firm growth and market structure.

Like previous theoretical models of firm innovation and growth (Klette & Griliches, 2000; Klette & Kortum, 2004), this paper seeks to evaluate a number of the stylized facts found in the empirical literature. However, our contribution is unique in that our model considers firm efficiency to be the driver of firm growth. Specifically, the stylized facts we wish to address are[2]:

(i) R&D expenditures rise proportionally with firm size and with R&D intensities independent of firm size

(ii) The number of patents and innovations per dollar of R&D tends to decrease with firm size

(iii) Persistent differences in firm sizes (Klette & Griliches, 2000)

(iv) Smaller firms tend to grow faster than larger firms, although among larger firms growth rates are unrelated to past growth or firm size.

The last stylized fact requires a few words. This is a refinement of Gibrat's law which held that firm growth and size are uncorrelated. However, recent empirical work rejects this hypothesis for very small firms. This is important because Gibrat's law is often imposed on theoretical models of firm growth (e.g. Klette & Griliches, 2000). Since firm growth is determined by firm efficiency in our model, it is desirable that our model is consistent with the analogous observation that smaller firms grow faster (Sutton, 1997) and are more efficient (Dhawan, 2001) than large firms. Our model reconciles this observation with Demsetz's hypothesis of large, efficient firms by considering separate

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[2] Detailed references & surveys behind these facts can be found in Appendix A of Klette & Kortum (2004).

measures of technical efficiency and allocative efficiency that firms may trade-off, with smaller firms likely to minimize their static, allocative inefficiency and larger firms likely to minimize their dynamic, technical inefficiency.

The rest of the paper is organized as follows. As the literatures on firm innovation and firm efficiency have largely developed along separate lines, we will briefly discuss the current thought on firm size, market structure, and innovation and then efficiency and market structure. This is followed by our endogenous model of firm innovation, efficiency, and growth. We conclude with an empirical test of the model using U.S. manufacturing firm data from the COMPUSTAT database.

II. Literature Review

i. Firm size, market structure, and innovation

Cohen & Klepper (1996) develop a model of firm innovation and firm size designed to address the stylized facts that: (i) R&D expenditures rise proportionally with firm size and with R&D intensities independent of firm size; and (ii) The number of patents and innovations per dollar of R&D tends to decrease with firm size. These empirical regularities have traditionally been interpreted to mean that large firms possess no advantage in R&D. In light of the apparent diminishing returns to R&D implied above, in seems peculiar that large firms would continue to perform R&D at relatively high nominal rates. Cohen & Klepper (1996) account for this discrepancy by developing a model where firm size conditions the returns to R&D by spreading the costs over the output of a given product. This cost spreading approach generates an advantage to R&D

for firms with large market shares but does not address the larger issue of firm growth and market structure.

Seeking to explain both questions of why dominant firms perform relatively more research and the persistence of monopoly, Etro (2004) develops a theoretical model of patent races under Stackleberg competition. Under the assumption of free entry, Etro finds that dominant firms must aggressively innovate to avoid market decline. Further, markets characterized by the continuous leapfrogging of new patent holders must possess some barriers to entry, suggesting that persistence of monopoly actually indicates competitive markets. While this result appears counter-intuitive, it is actually in line with Demsetz's (1973) idea that dominant firms are far from basking in the quiet life, but earn their dominance through superior performance, which we append are maintained by continual innovation. Empirically testing whether dominant firms have greater incentives to innovate, Blundell *et. al.* (1999) find that dominant firms who innovate actually receive higher stock market valuations than those who do not, which could account for their high rate of innovation despite decreasing returns per dollar of R&D.

Borrowing from the quality ladder models of macroeconomic growth, Klette and Griliches (2000) develop a endogenous model of firm growth where firms receive stochastic innovations due to R&D investment. Firm growth is dependent upon the receipt of monopoly profits until the next innovation arrives. This generates results consistent with Gibrat's law as well as the stylized facts presented previously. Klette and Kortum (2004) extend this model to a general equilibrium framework and by assuming that firm innovations accumulate into its stock of knowledge capital. Additional implications of their model are that R&D intensity is positively related to productivity and that larger firms can offset diminishing returns to R&D investment through ever increasing stocks of knowledge capital.

ii. Concentration, market power, and efficiency

This literature begins with Demsetz (1973), who posits that industry concentration may occur because of firm efficiency. The rationale is that firm specific effects, such as team production or reputation, may exist which can not be obtained by deconcentration in an industry. Segregating firms from the U.S. Census of Manufactures into strategic groups based upon their market shares, Martin (1988) states that efficiency and market power may not be competing theories of industry concentration, finding evidence that both effects are present in his data. Developing a new empirical industrial organization (NEIO) model that separates market power from efficiency effects, Lopez *et. al.* (2002) examine the effects of concentration in U.S. food processing industries. Their results are mixed, with one-third of their sample experiencing cost efficiency effects from concentration and the balance being dominated by market power effects that reinforce cost inefficiency. They note, however, that only industries characterized by product homogeneity and large economies of size experience such cost efficiencies in their sample.

A theoretical paper on efficiency and market structure, Pires and Brito (2003) demonstrates with models of Cournot-Nash competition and product differentiation where efficient firms begin with larger market shares, that it is possible for changes in efficiency to generate inverse changes in market share, particularly in the presence of dominant firms. This implies that whether the Demsetz hypothesis holds or not, this hypothesis can not be extended to variations in efficiency.

III. A Simple Model of Endogenous Firm Innovation

i. Basic model

With regards to the literature, our model is closest to that of Cohen & Klepper (1996), although we employ a stochastic, rather than deterministic innovation process. A significant departure from previous efforts is our assumption regarding the body of knowledge firms acquire over time. Prior to Klette & Kortum (2004), models have relied upon flows of R&D investment to generate new innovations. Like Klette & Kortum (2004), we assume that firms carry a stock of knowledge over time, however, we allow all firms to utilize the updated aggregate body of knowledge in periods subsequent to the arrival of new innovations. This is due to another key departure from the literature in our model.

Current models of endogenous firm innovation are variations of the quality ladder models utilized in macroeconomic growth (Grossman & Helpman, 1991; Aghion & Howitt, 1992). These variations specify that new innovations are *drastic*[3] in nature, with firm growth driven by the monopoly profits received by the innovating firm until the stochastic arrival of the next innovation. However, as Mansfield *et. al.* (1981) note, many patents are either successfully imitated, or imitated around, within 1 year and most

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[3] Arrow (1962) defines *drastic* innovations as those that completely supplant existing products or technology and *non-drastic* innovations as those that are constrained by competition from previous products or technology. When *non-drastic* innovations are considered in the literature (e.g. Klette & Griliches, 2000), competitors are still driven out of the market by the setting of a limit price on the part of the monopolist.

in less than 3 years. Further, this interpretation of innovation is largely focused upon the creation of distinctly new products that can supplant previous products in the market.

Our model of innovation primarily considers the development of process improvements and new technology that is applied in the production process. Firms are hardly inefficient because they fail to produce products enjoying patent protection. However, success or failure in the discovery and utilization of new, innovative manufacturing technologies, for example, standardized parts, assembly lines, Just-in-Time (JIT) inventory systems, etc., may often dictate the growth and performance of firms, particularly in competitive industries. These kinds of innovations are often difficult to protect through formal patents, generating technology spillovers that may quickly diffuse, especially in industries with high levels of turnover and labor mobility.

Coupling this interpretation of innovation with the empirically observed short effective life of patents, we follow Cohen and Klepper (1996), and limit an innovator's monopoly return to new innovation to one period before updating the aggregate body of knowledge available to all firms. One practical implication of this assumption is that no firm possesses an advantage in innovation over other firms, except with regards to their physical capability to invest in R&D. An additional unique characteristic of our model is that given our assumption of *non-drastic* innovations, all firms may simultaneously receive innovations at a given point in time. It is this stochastic innovation process that allows us to define measures of firm efficiency that drive firm growth and ultimately market structure.

Consider a model of firm production, where a given firm i at time t produces output, Y_{it}, with its input endowments of capital K_{it} and labor L_{it}:

(1) $$Y_{it} = A_{it} F(K_{it,} L_{it}) .$$

A_{it} represents an index of the technology that a given firm can bring to bear on its resource endowments in the production process. Employing a Cobb-Douglas functional form for convenience[4], equation (1) can be rewritten as:

(2) $$Y_{it} = A_{it} K_{it}^{a} L_{it}^{1-a} .$$

An improvement in the technology available to a firm in time t is a stochastic process that is governed by the research efforts of the firm. We assume that a firm can undertake research to develop superior technology in the next period by investing resources to obtain an instantaneous probability of innovation p, with $0 \le p \le 1$. The instantaneous probability of innovation is directly related with the resources allocated to research, $K_{it}^{r} \in (0, K_{it})$ and $L_{it}^{r} \in (0, L_{it})$:

(3) $$p_{it} = F(K_{it-1}^{r}, L_{it-1}^{r}) ,$$

with $p_{it}(0,0) = 0$, $\frac{\partial p_{it}}{\partial K_{it-1}^{r}}, \frac{\partial p_{it}}{\partial L_{it-1}^{r}} > 0$, and $\frac{\partial^2 p_{it}}{\partial {K_{it-1}^{r}}^2}, \frac{\partial^2 p_{it}}{\partial {L_{it-1}^{r}}^2} < 0$. Advances in the technology index are then discrete random variables Z_{it} where $P(Z_{it} = z_t) = p_{it}$, $P(Z_{it} = 0) = (1 - p_{it})$, and with z_t representing the value of the new innovation's improvement on the technology index. We shall employ Z_{it}^{*} to denote the realization of random variable Z_{it}. Given the assumption of *drastic* innovations in much of the literature, it is usually important to carefully consider the timing before the next innovation, with researchers typically utilizing either a Poisson (e.g. Klette & Griliches,

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[4] The assumption of a Cobb-Douglas functional form is for expositional ease and is not necessary for the arguments to be developed in this model.

2000; Etro, 2004; Klette & Kortum, 2004) or exponential (Reinganum, 1989) distribution for the probability of an innovation arriving within a specific interval. Since our model, like Cohen and Klepper (1996), relies upon *non-drastic* innovations that are universally disseminated the period after arrival, the specific probability distribution is not crucial beyond standard assumptions that it be continuously twice differentiable.

As all firms freely obtain the new innovation in the subsequent period, increases in the technology index does not convey an advantage in research to any particular firm. Though we could consider the probability of a new innovation to be conditional upon the ever increasing index of technology, this would not vary between firms, which face the same aggregate technology index each period. Since we desire to examine firm heterogeneity in R&D and market share, we assume that future innovations are entirely determined by present allocations of resources to R&D. Thus, in period t the expected technology index for the ith firm is:

$$A_{it} = E[Z_{it}] + A_{t-1} = p_{it}(K^r_{it-1}, L^r_{it-1})z_t + A_{t-1} \tag{4}$$

or more explicitly,

$$A_{it} = p_{it}(K^r_{it-1}, L^r_{it-1})z_t + \sum_{s=1}^{t-1}\sum_{j=1}^{n} Z^*_{js}, \tag{5}$$

with A_{t-1} capturing the aggregate body of knowledge available to all firms in time t. Substituting equation (5) into equation (2), we produce the following model for the ith firm's expected production in time t:

$$Y_{it} = \left(p_{it}(K^r_{it-1}, L^r_{it-1})z_t + \sum_{s=1}^{t-1}\sum_{j=1}^{n} Z^*_{js} \right)(K_{it} - K^r_{it})^a (L_{it} - L^r_{it})^{1-a}. \tag{6}$$

Assuming that output markets can be initially characterized by perfect competition, each firm's objective function is simply (7), with each firm choosing (K_{it}^{r}, L_{it}^{r}):

$$\max_{K_{it}^{r}, L_{it}^{r}} \sum_{t} Y_{it} \mathbf{d}^{t} \tag{7}$$

where d represents the discount rate for future production.

ii. Innovation and Firm Efficiency

For any time t the maximum attainable output, Y_{it}^{F}, of each firm, given their resource endowment, (K_{it}, L_{it}), is $Y_{it}^{F} = A_{t} K_{it}^{a} L_{it}^{1-a}$, where

$$A_{t} = \sum_{j=1}^{n} Z_{it}^{*} + \sum_{s=1}^{t-1} \sum_{j=1}^{n} Z_{js}^{*} . \tag{8}$$

The first sum captures the probability of the ith firm receiving an innovation during time t and the subsequent summations, again, capture the aggregate value of previous innovations. This yields each firm's boundary of production,

$$Y_{it}^{F} = \left(\sum_{j=1}^{n} Z_{it}^{*} + \sum_{s=1}^{t-1} \sum_{j=1}^{n} Z_{js}^{*} \right) (K_{it} - K_{it}^{r})^{a} (L_{it} - L_{it}^{r})^{1-a} . \tag{9}$$

However, a given firm may not produce at this boundary due to the idiosyncratic arrivals of innovation and/or a firm's decision to undertake research, which govern the innovation process. We define measures of firm inefficiency that incorporate these possibilities as follows. Considering, again, the ith firm's boundary of production:

$$Y_{it}^{F} = A_{t} K_{it}^{a} L_{it}^{1-a} . \tag{10}$$

The actual production of the ith firm in time t is:

$$\begin{array}{ccccccc} & & \textit{potential output} & \times & \textit{technical efficiency} & \times & \textit{allocative efficiency} \\ (11) \quad Y_{it} & = & A_t K_{it}^{a} L_{it}^{1-a} & \times & \dfrac{A_{it} K_{it}^{a} L_{it}^{1-a}}{A_t K_{it}^{a} L_{it}^{1-a}} & \times & \dfrac{A_{it}(K_{it} - K_{it}^{r})^{a} (L_{it} - L_{it}^{r})^{1-a}}{A_{it} K_{it}^{a} L_{it}^{1-a}} \end{array}$$

Rewriting (11) yields:

$$\begin{aligned} Y_{it} &= A_t K_{it}^{a} L_{it}^{1-a} \times \frac{A_{it}}{A_t} \times \frac{(K_{it} - K_{it}^{r})^{a} (L_{it} - L_{it}^{r})^{1-a}}{K_{it}^{a} L_{it}^{1-a}} \\ &= A_{it}(K_{it} - K_{it}^{r})^{a} (L_{it} - L_{it}^{r})^{1-a} \end{aligned} \qquad (12)$$

which is the quantity of output produced using the firm's available technology in time t, as opposed to the best available technology, and productive resources not dedicated towards research. We formally define our measures of firm inefficiency as:

$$(13) \qquad \frac{A_{it}}{A_t} \equiv \frac{1}{u_{itz}} \in (0,1)$$

and

$$(14) \qquad \frac{(K_{it} - K_{it}^{r})^{a} (L_{it} - L_{it}^{r})^{1-a}}{K_{it}^{a} L_{it}^{1-a}} \equiv \frac{1}{u_{itx^r}} \in (0,1) .$$

u_{itz} is a measure of technical inefficiency equivalent to the potential output lost when a firm does not receive an innovation nor can exploit the innovations of other firms[5]. Because our model assumes simultaneous non-drastic innovations, a firm will always possess some technical inefficiency in this model unless no other firm receives an innovation or no firm receives an innovation at a given time t, in which case every firm produces with the current body of knowledge, A_{t-1}. The other inefficiency identified in

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[5] Recall that we assume all firms may simultaneously receive non-drastic innovations which are included in the body of knowledge in time $t+1$.

our model, u_{itx^r}, is a measure of allocative inefficiency which is equal to the production lost in time t by allocating resources to research. Because firms may maximize their output over time by undertaking research to improve their productive capacity[6], this measure of allocative inefficiency is only true in a static sense. However, it provides a plausible explanation that may link some of the observed stylized facts, such as larger firms investing more heavily in R&D, small firms being observed to produce with greater efficiency, and with the persistence of larger firms' market shares, nonetheless.

In short, consistent with Martin's (1988) findings, sloth and efficiency may not be mutually exclusive states for dominant firms. Dominant firms may be efficient as Demsetz (1973) suggests, albeit dynamically through their investment in future productive potential, and yet vulnerable, inefficient targets for small and lean producers at a given point in time. These smaller, efficient firms must then quickly steal away market share before the next innovation strikes, always leaving them a step behind due to their technical inefficiency, u_{itz}. Of course, as they survive and grow, they too may trade off some u_{itx^r} to reduce u_{itz} depending upon the specific distribution of $p(K_{it}^{r}, L_{it}^{r})$ and the value of z_t. Formally including our measures of technical and allocative efficiency into a single model of firm production produces:

$$(15) \qquad Y_{it} = A_t K_{it}^{a} L_{it}^{1-a} u_{itz}^{-1} u_{itx^r}^{-1}.$$

In the next section we use this model to investigate how the innovation process drives firm growth and ultimately market structure.

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[6] This depends on the actual distribution of $p(K_{it}^{r}, L_{it}^{r})$ and the value of z_t

IV. An Analysis of Firm Growth and Market Structure

Before we can examine how our model of firm innovation and efficiency may affect market structure, we must explicitly incorporate firm growth into the model. Suppose an industry consists of n identical firms endowed with resources (K_0, L_0). The initial index of technology available to all firms is A_0. Analogous to Cohen and Klepper (1996), we will simplify our model by assuming that some fixed proportion[7] g of the value of firm output is retained to acquire resources for further production:

$$(16) \qquad R_t K_{it} + W_t L_{it} = gP_{t-1}Y_{it-1},$$

where R_t, W_t, and P_{t-1} are the market prices of capital, labor, and output, respectively, and with firms acquiring additional capital K_{it} and labor L_{it} such that $R_t \frac{\partial Y_{it}}{\partial K_{it}} = W_t \frac{\partial Y_{it}}{\partial L_{it}}$. We will denote c_t as the proportion of firm revenues $gP_{t-1}Y_{it-1}$ that is used to acquire additional capital inputs in time t as, with the remainder, $(1-c_t)$, being allocated toward additional labor inputs[8]. Assuming that the distribution of $p(K_{it}^r, L_{it}^r)$ and the value of z_t are sufficient to induce firms to innovate, each firm allocates a portion of firm resources towards R&D investment: $K_{it}^r = k_{it}K_{it}$ and $L_{it}^r = l_{it}L_{it}$, with $k_{it}, l_{it} \in (0,1)$. Firms grow through an iterative process of producing ever larger quantities of output which generates ever larger revenues from which to acquire more resources for production.

Because we are considering identical firms in perfectly competitive markets, if we momentarily suspend the possibility of firm inefficiency, A_{it}, k_{it}, and l_{it} are reduced to

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[7] Allowing the portion of firm revenues to vary over time and firm are obvious extensions to this model.

[8] In theory, each firm's initial factor endowments came from a similar budget allocation of seed capital.

A_t, k_t, and l_t, respectively. Suppressing factor and output prices, the output of firm i in time 0 and time 1 are, respectively:

$$(17) \qquad Y_{i0} = A_0 (K_0 - k_0 K_0)^a (L_0 - l_0 L_0)^{1-a} = A_0 K_0^a L_0^{1-a} (1-k_0)^a (1-l_0)^{1-a}$$

$$(18) \qquad \begin{aligned} Y_{i1} &= A_1 (c_1 g Y_{i0})^a ((1-c_1) g Y_{i0})^{1-a} (1-k_1)^a (1-l_1)^{1-a} \\ &= g K_0^a L_0^{1-a} A_0 A_1 (1-k_0)^a (1-l_0)^{1-a} (1-k_1)^a (1-l_1)^{1-a} (1-c_1)^a (1-c_1)^{1-a} \end{aligned} .$$

Solving through iteration yields the ith firm's output in time t:

$$(19) \qquad Y_{it} = g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a}$$

and its market share:

$$(20) \qquad MS_{it} = \frac{g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a}}{\sum_{i=1}^{n} \left[g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right]} .$$

Thus, if we assume away the stochastic innovation process embedded in A_m, each firm comprises a $1/n$ share of the market.

Returning our stochastic innovation process to the model we can examine how firm efficiency influences firm growth and market structure. Retaining our assumption of an industry of n identical firms, we begin with each firm allocating the same proportion k_t and l_t of capital and labor resources to R&D investment. The ith firm's output in time t is now:

$$(21) \qquad Y_{it} = g^t K_0^a L_0^{1-a} A_{it} \prod_{l=0}^{t-1} A_l \prod_{m=0}^{t} c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} ,$$

or alternatively,

$$(22) \qquad Y_{it} = g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1}$$

and its market share:

(23)
$$MS_{it} = \frac{g^t K_0^a L_0^{1-a} A_{it} \prod_{l=0}^{t-1} A_l \prod_{m=0}^{t} c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a}}{\sum_{i=1}^{n} \left(g^t K_0^a L_0^{1-a} A_{it} \prod_{l=0}^{t-1} A_l \prod_{m=0}^{t} c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right)} = \frac{g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1}}{\sum_{i=1}^{n} \left(g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1} \right)}.$$

As previously discussed, even when all firms receive an innovation they are inefficient relative to the aggregate index of technology they could potentially bear on their resources and *will get to bear on their resources in the next period*. However, if all firms, or no firms, receive innovation z_t, i.e. $u_{itz} = u_{jtz} \, \forall \, i, j$, their respective market shares remain unchanged with each firm receiving an even share of the market.

Now let's consider that firm i does not receive an innovation with the remaining j firms such that $u_{itz} > u_{jtz}$. Suppressing the u_{jtz} term of the innovative firms to ease our exposition, the market share of firm i is now:

(24)
$$MS_{it} = \frac{g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1}}{\sum_{j=1}^{n-1} \left(g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] \right) + g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1}}$$

with the market share of the remaining firms:

(25)

$$MS_{jt} = \frac{g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right]}{\sum_{j=1}^{n-1} \left(g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] \right) + g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1}}.$$

Assuming that all firms receive innovations each period from this point forward[9], the deviation in market shares will persist, despite the fact that all firms will exploit the exact same technology in subsequent periods. This is because of firm i's diminished resource base from which to produce or conduct research. The larger firms now have an enhanced incentive to undertake research, since each innovation can be brought to bear on a much larger resource base[10]. However, small firms may be able to recapture these losses by reallocating resources from research to production and thus reducing their static allocative inefficiency u_{itr}[11].

V. An Empirical Evaluation of the Model

Though we have presented a model of endogenous firm growth that defines firm efficiency as a tradeoff between a static, allocative efficiency and a dynamic, technical efficiency, the validity of this tradeoff and the degree to which these effects off-set or dominate one another is an empirical question. For example, considering a firm's market share as produced by the model:

$$(26) \qquad MS_{it} = \frac{g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1} u_{itx^r}^{-1}}{\sum_{i=1}^{n} \left(g^t K_0^a L_0^{1-a} \prod_{m=0}^{t} \left[A_m c_m^a (1-c_m)^{1-a} (1-k_m)^a (1-l_m)^{1-a} \right] u_{itz}^{-1} u_{itx^r}^{-1} \right)},$$

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[9] And that firm i's optimal resource allocation hasn't changed due to its diminished market share, although doubtless it would. Exploration of this issue through formal simulation shall be explored in future research.

[10] This is analogous to Cohen & Klepper's cost spreading (1996).

[11] An evaluation of the changes in market structure because of u_{itr} is analogous to that produced by u_{itz} and is not formally presented due to its dependency on the distribution of $p(X_{it}^r)$ and the value of z_{it}.

we would expect efficient firms to grow in size and inefficient firms to shrink relative to the overall market. However, firm decisions to reduce the static allocative inefficiency u_{itx^r} would increase the expected dynamic technical inefficiency u_{itz}, since it is investments in R&D today that generate tomorrow's technological progress.

We are able to test this theoretical framework by estimating a stochastic frontier model that can explicitly estimate a firm's efficiency relative to the technology frontier and examine determinants of the inefficiency effect. This technique is especially beneficial for our analysis as it allows us to incorporate R&D investments as determinants of efficiency as opposed to inputs to a production function, where R&D expenditures have been shown to be an unreliable instrument for evaluating the relationship between innovation and market share (Blundell *et. al.*, 1999). Estimating a stochastic frontier with U.S. manufacturing firm data from Standard and Poor's COMPUSTAT database, we find firm market shares and firm size to be highly, positively correlated with firm efficiency and research intensity to decrease firm efficiency, as predicted by the theoretical model.

Our estimates of firm efficiency are also consistent with previous studies that have estimated stochastic frontiers with COMPUSTAT data. Our empirical analysis is closest to Baek (2004), who utilized this approach to determine whether international diversification was related to a firm's productive efficiency. Other similar studies include Dhawan (1997), who applied a stochastic frontier model to evaluate the technical progress of firms in the COMPUSTAT database and favorably compared the results to those obtained from traditional estimates of Solow residuals.

i. *Method*

We evaluate the relationships between a firm's size, market dominance, R&D strategy, and productive efficiency implied by equations (15) and (26) by employing the stochastic frontier model proposed by Battese & Coelli (1995). Stochastic frontier models, first introduced by Aigner *et al* (1977) and Meeusen & van den Broeck (1977)[12], estimate production frontiers using a composite error component that directly estimates a firm's inefficiency relative to frontier while also accounting for random noise. The Battese & Coelli (1995) formulation accommodates panel data and also allows for the evaluation of the determinants of technical inefficiency:

(27) $\ln Y_{it} = \ln f(X_{it}; b_t) + v_{it} - u_{it}$

where Y_{it} is the output for firm i in the t th period; $f(X_{it}; b_t)$ is a deterministic function of a vector of input quantities, X_{it}, and a vector of unknown parameters, b_t; v_{it} are independent and identically distributed $(i.i.d.)$ random errors with a $N(0, s_V^2)$ distribution; u_{it} are independently, but not identically, distributed non-negative truncated normal errors, with mean m_u and variance s_u^2, associated with inefficiency in production.

Rewriting the firm production function (Equation 15) derived from our model of stochastic innovation and firm efficiency in logarithms yields:

(28) $\ln Y_{it} = \ln A_t + a \ln K_{it} + (1-a) \ln L_{it} - \ln u_{itz} - \ln u_{itx^r}$.

As equation (27) shows, our model of firm efficiency can be directly estimated through the Battese and Coelli (1995) framework, although the estimates for firm inefficiency,

(29) $\ln u_{it} = \ln u_{itz} + \ln u_{itx^r}$,

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[12] A comprehensive treatment is Kumbhakar and Lovell (2000)

capture total firm inefficiency. However, the Battese and Coelli (1995) model allows us to estimate determinants of firm inefficiency. Since u_{itx^r} only represents firm inefficiency resulting from the allocation of productive resources to research and development, we can directly estimate the effects of firm R&D on the total inefficiency term u_{it}. We interpret the unexplained portions of the inefficiency term as representing the technical inefficiency defined by u_{itz}.

To examine explanations of firm efficiency, the efficiency effect, u_{it}, can be specified as:

(30) $$u_{it} = z_{it}d + w_{it}$$

where z_{it} is a vector of observable explanatory variables; d is the vector of parameters to be estimated; and $w_{it} \sim N(0, s_w^2)$ with the distribution of w_{it} being bounded by the variable truncation point $-z_{it}d$.

Equations (28) and (30) are jointly estimated through maximum likelihood, with the likelihood function maximized in terms of the variance parameters, $s^2 = s_v^2 + s_u^2$ and $g = s_u^2 / s^2$. Estimates of the efficiency of the ith firm in time t is given by

(31) $$TE_i = \exp(-u_{it}) = \exp(-z_{it}d - w_{it}),$$

which is interpreted as the degree which output falls short of its potential given a specific technology and level of inputs. It can be predicted by

(32) $$E[\exp(-u_{it}) \mid (v_{it} - u_{it})] = \left[\exp\left(-u_{it}^* + \frac{1}{2}s^{*2}\right)\right] \times \left[\frac{\Phi\left[\left(u_{it}^*/s^*\right) - s^*\right]}{\Phi\left(u_{it}^*/s^*\right)}\right]$$

where $u_{it}^* = \dfrac{s_v^2(z_{it}d) - s_u^2(w_{it})}{s_v^2 + s_u^2}$ and $s^{*2} = \dfrac{s_v^2 s_u^2}{(s_v^2 + s_u^2)}$.

Using firm level data, we will estimate the following Cobb-Douglas production function:

$$(33) \qquad \ln Y_{it} = b_0 + b_1 \ln K_{it} + b_2 \ln L_{it} + a_1 t + a_2 SMALL + a_3 I_{it} + v_{it} - u_{it}$$

where Y_{it} is the value added of production for firm i in time t; L_{it} is the number of employees; and K_{it} is the value of fixed assets. $SMALL_{it}$, I_{it}, and t are indicator variables to control for firm size, industry effects and time, respectively. Following our hypothesis about the relationships between a firm's size, research efforts, market dominance and its efficiency, the z_{it} variables we will use to examine firm efficiency are: $SMALL_{it}$, an indicator variable for the size of a given firm; MS_{it}, a firm's market share; RDS_{it}, a firm's R&D intensity; and RDI_{it}, a firm's R&D intensity relative to their industry. As previously discussed, the estimates of RDS_{it} and RDI_{it} capture the portion of u_{it} accounted for by u_{itx^r} with the remainder attributed to technical inefficiency u_{itz}.

ii. Data

We utilize a balanced panel of firm level data extracted from Standard and Poor's COMPUSTAT database for the period of 1994-2003. The panel covers all active and inactive manufacturing firms, as identified by 1-digit Standard Industrial Classification (SIC) Codes 2 & 3, that were listed on the New York, American, NASDAQ, and regional stock exchanges during this period. The raw unbalanced panel contains 108,208 observations for the period of 1994-2003 and spanning 6,763 firms, due to the appearance and disappearance of firms at different years in the panel. After removing observations with variables for which no data was reported, we were left with an unbalanced panel of 25,584 observations spanning 3,550 firms. This was further reduced to a balanced panel of 6,920 observations, covering 692 firms from 1994-2003. While a longer, albeit thinner, panel could have been used, we feel that this analysis benefits from the increased firm heterogeneity that a wider panel provides. We note that any variables calculated from the data were calculated using the full raw panel of 108,208 observations. Therefore, despite their omission, we are able to account for some degree of the variance these additional observations may have exerted in our analysis.

Value-added, defined as net sales less the cost of goods sold plus the change in inventory, is employed for each firm's output. Value-added was converted into real 2000 dollars using the GDP deflator. Labor inputs are defined as the number of employees as reported to shareholders each year. We use fixed assets, defined as the market value of total assets less current assets, as our measure of capital. Following standard practice, the reported book values are converted into market values using the Salinger and Summer (1983) perpetual inventory method described in Whited (1992). Finally, we deflate our measure of capital stock with the Producers Price Index.

Each firm's market share in time t is defined as firm sales divided by total industry sales, with each industry defined by 3-digit SIC codes. We utilize two different measures of R&D: the ratio of reported R&D expenses to firm sales and firm R&D intensity relative to industry, defined as each firm's R&D to sales ratio divided by the industry's R&D to sales ratio. R&D intensities greater (less) than 1 indicate that a firm commits relatively more (less) resources to R&D than the industry as a whole, after controlling for firm size. Industry effects are controlled through indicator variables for the 83 3-digit SIC codes represented in our data. Figure 1 presents the distribution of firms in each industry.

[Insert Table 1: Distribution of Firms by Industry]

Finally, indicators are employed to denote small and large firms, using Gertler & Gilchrist's (1994) definition of small firms as those with average assets of less than $25 million in 1982 dollars[13]. 2,730 observations or 39% of the sample are classified as small firms by this definition. Summary statistics for balanced panel are presented below in Table 1.

[Insert Table 1. Summary Statistics of the Variables]

Figure 2 shows a series of scatter-plots between ln(output), market share, R&D to sales ratio, and firm R&D intensity relative to industry. These echo common regularities found in firm data: (i) there is no relationship between firm size and research efforts; (ii) no relationship between a firm's market share and R&D; and (iii) no relation between a firm's R&D intensity and their relative R&D intensity with respect to their industry[14]. Figures 3 & 4 present the same scatter-plots for the small and large sub-samples and reveal that these findings are consistent for both groups.

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[13] While this is an arbitrary definition, Dhawan (2001) found no qualitative difference in results when using cut-off levels of $35 million, $50 million, and $100 million to examine firm size and productivity differentials.

[14] We find these patterns are robust to alternative measures of firm size, such as sales, assets, and employees. We also find nominal R&D expenditures to be positively related to firm size.

[Insert Figure 2. Scatter-plot of Select Variables (All Firms)]

[Insert Figure 3. Scatter-plot of Select Variables (Small Firms)]

[Insert Figure 4. Figure 4. Scatter-plot of Select Variables (Big Firms)]

iii. Results

Table 2 presents the estimates for the stochastic frontier model in equation (28). The estimate for g , which is calculated as $g = \frac{s_u^2}{s_u^2 + s_v^2}$, indicates to what degree deviations from the frontier can be attributed to inefficiency as opposed to statistical noise. The null hypothesis that g equals zero is rejected suggesting that inefficiency is present within the data. The average technical efficiency score of the sample is .685 which is in the range of estimated average score efficiency scores, .627 - .724, Baek (2004) received with similar COMPUSTAT data.

[Insert Table 2. Maximum Likelihood Estimates for Firm Model]

Table 3 presents our estimates with the inclusion of explanatory variables z_{it} for the technical inefficiency term $\exp(-u_{it})$. We find Model 1.5 to dominate the other model specifications through log-likelihood testing. Some care must be taken in the interpretation of the estimated z_{it} s for the inefficiency term u_{it}. Since u_{it} represents deviations from the efficient frontier, a positive (negative) estimated coefficient indicates that this explanatory factor decreases (increases) the efficiency of the firm.

[Insert Table 3. Maximum Likelihood Estimates of Inefficiency Effects]

As Table 3 reveals, the estimates for capital K and labor L are quite close regardless of model specification. Although the indicator for small firms has a highly significant, negative effect on output when it is not considered a determinant of firm inefficiency, we find it to be a highly significant determinant of firm inefficiency. Since $SMALL$ has a positive, yet statistically insignificant effect on output when the indicator is employed in both the production function and as a determinant for firm inefficiency, this implies that any negative influence firm size may have exerted on firm output in Models 1.0 – 1.2 is captured under the inefficiency effect[15]. We can interpret this to mean that firm size has little effect on firm production and that larger firms are characterized by greater efficiency, a prediction of our model.

While we can not formally decompose the total inefficiency estimate u_{it} into its comprising technical efficiency u_{itz} and allocative inefficiency u_{itx^r}, our estimate of RDS identifies the degree to which u_{itx^r} influences total inefficiency u_{it}. As our model predicts, allocating resources to research is a highly significant contributor to total firm inefficiency. However, the magnitude of this estimate is dwarfed by the role of market share on firm efficiency. Since the remaining inefficiency not explained by firm investment in R&D can be attributed to sheer technical inefficiency u_{itz}, we can see that

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[15] Model specifications that did not include the indicator variable for small firms in the production function were all rejected through log-likelihood testing and are not reported.

technical inefficiency plays a much larger role in firm production that the allocation of resources to research.

We can also observe evidence of another important implication of our model in our results. Our model of firm innovation implies that technical inefficiency and allocative inefficiency can also be thought of as a tradeoff between dynamic (technical) and static (allocative) efficiency. A common stylized fact (Klette & Kortum, 2004) is that R&D expenditures increase proportionally with firm size, despite the presence of decreasing returns to R&D. As our results indicate that firm efficiency increases with firm size and decreases with investments in R&D, which grow proportionally with firm size, this would provide some evidence that larger firms are indeed trading off static efficiency for dynamic efficiency, while smaller firms focus on near term increases in output as they attempt quickly to grow and survive.

VI. Conclusion

We have presented a theoretical model of endogenous firm growth that conforms to empirical regularities found in the literature and explicitly defines firm efficiency as the principal driver of firm growth and market structure. While our analysis of market structure was restricted to a highly stylized case of identical firms, the model can be easily extended to accommodate heterogeneous assumptions of the innovation process, innovation values, and initial market shares. In particular, extending the model to permit firm entry and exit should provide a richer set of conclusions that better emulate what is observed in the data. Testing the predictions of this model by estimating a panel stochastic frontier with U.S. manufacturing data, we find that a firm's market share, size, and R&D intensity are highly significant as determinants of a firm's efficiency. Specifically, we find that firm efficiency increases with higher market shares. It is clear from our results that firms which undertake higher levels of R&D exhibit a significant inefficiency effect, which is consistent with then notion that research detracts resources from productive activities. This supports the theoretical model's prediction that firms face a tradeoff between dynamic (technical) and static (allocative) efficiency. Further research is required to capture the dynamic efficiency gains that firms should receive from their sacrifice of static efficiency and the formal decomposition of these two effects from estimates of total firm efficiency.

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Figure 1. Distribution of Firms by Industry

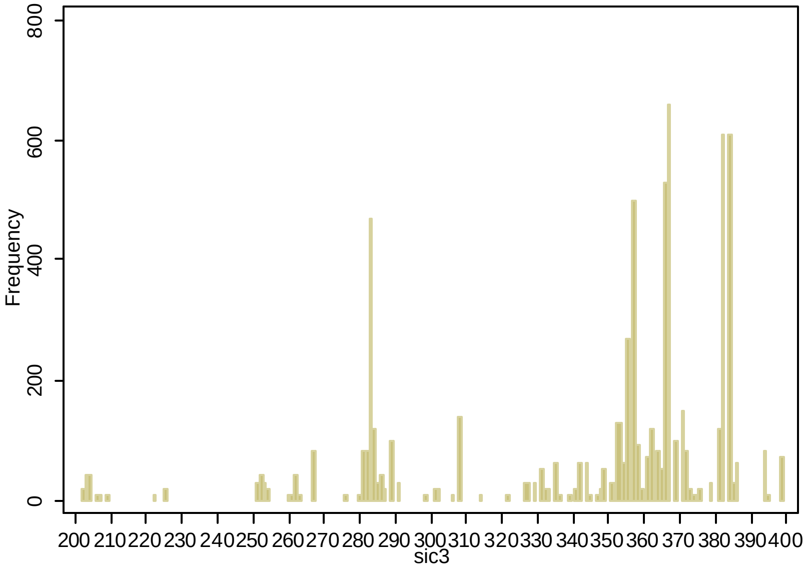

Figure 2. Scatter-plot of Select Variables (All Firms)

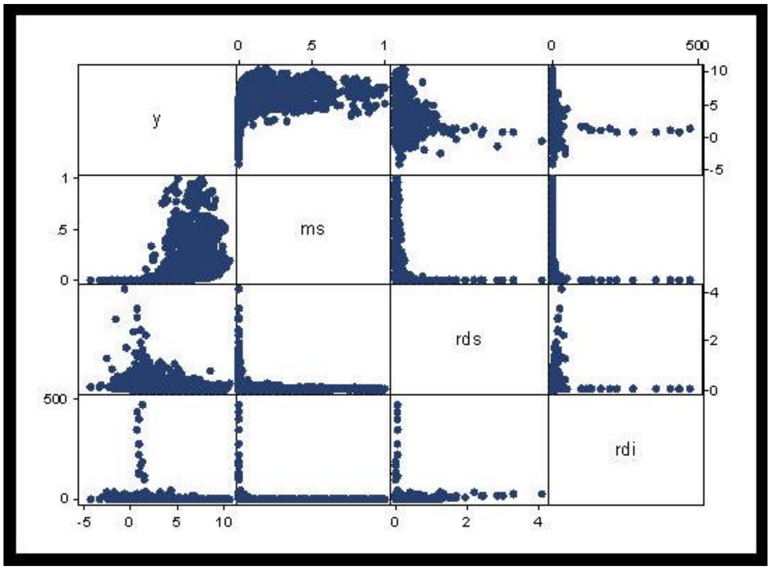

Figure 3. Scatter-plot of Select Variables (Small Firms)

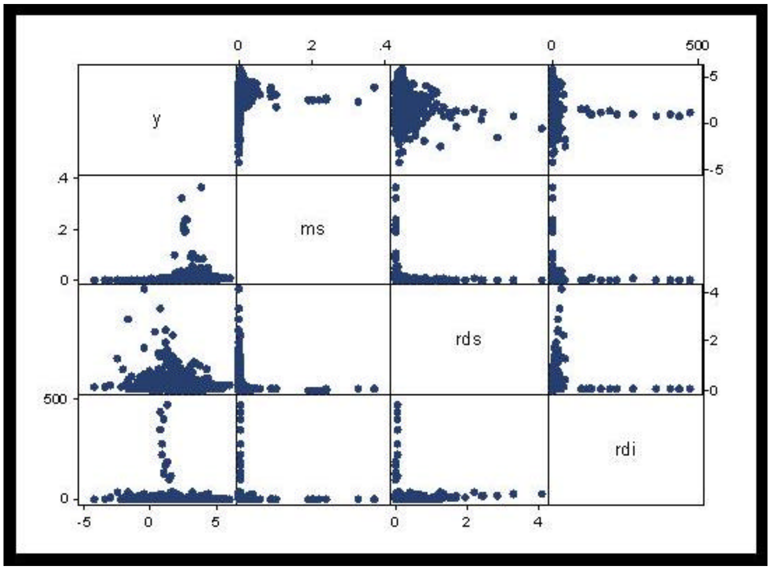

Figure 4. Scatter-plot of Select Variables (Big Firms)

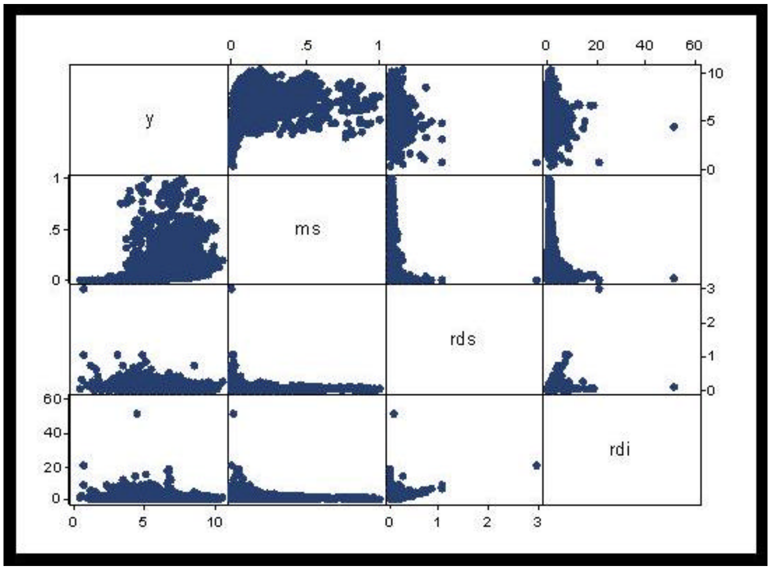

Table 1. Summary Statistics of the Variables

Variable	Sample Mean	Standard Deviation	Minimum	Maximum
Value-Added (Y)	758.662	2427.848	.015	37542.64
Fixed Assets (K)	1298.243	5477.028	.008	119025.1
Employees (L)	8.218	26.866	.002	728
Market Share (MS)	.061	.144	0	1
R&D to Sales (RDS)	.081	.148	0	4.116
R&D Intensity (RDI)	1.932	11.955	.005	472.343

Notes: 6920 Observations (t=10, n=692)

Table 2. Maximum Likelihood Estimates for Firm Stochastic Frontier Model

Variable	Coefficient (Std Error)	Variable	Coefficient (Std Error)
K	.334*** (.009)	s^2	.831*** (.050)
L	.591*** (.011)	s_v^2	.163*** (.006)
SMALL	-.153*** (.023)	s_u^2	.668*** (.052)
Constant	3.240*** (.083)	?	.803*** (.015)
Num Obs.	6920	Avg. TE score: .685	
Log-likelihood	-5666.791		
H_O: ?=0			
Test Statistic	862.075		
Decision	Reject		

Notes: *t*-ratios are asymptotic. *** $p < .001$; ** $p < .05$; * $p < .1$

Table 3. Maximum Likelihood Estimates of Inefficiency Effects

Variable	Model 1.0	Model 1.1	Model 1.2	Model 1.3	Model 1.4	Model 1.5
K	.338*** (.009)	.345*** (.010)	.349*** (.009)	.370*** (.010)	.363*** (.011)	.369*** (.053)
L	.488*** (.012)	.483*** (.019)	.475*** (.012)	.459*** (.013)	.471*** (.015)	.461*** (.073)
SMALL	-.163*** (.023)	-.160*** (.023)	-.150*** (.023)	.192** (.069)	.257** (.106)	.221 (.643)
Constant	3.208*** (.073)	3.091*** (.073)	2.990*** (.071)	2.954*** (.074)	2.915*** (.000)	2.985*** (.000)
μ:						
Constant	.681*** (.053)	.536** (.247)	.599*** (.053)	.467*** (.076)	.578*** (.069)	.583*** (.327)
MS	-29.274*** (1.877)	-27.817*** (2.918)	-28.137*** (1.761)	-27.095*** (1.923)	-26.065*** (2.338)	-27.308*** (7.203)
RDS		.706*** (.117)	.645*** (.077)	.582*** (.072)		.573*** (.125)
RDI			.001 (.001)	.001 (.001)		
SMALL				.600*** (.084)	.669*** (.111)	.564*** (.662)
s^2	.666*** (.025)	.620*** (.123)	.627*** (.021)	.580*** (.027)	.543*** (.035)	.578*** (.212)
s_v^2	.084*** (.004)	.094*** (.007)	.092*** (.004)	.080*** (.005)	.077*** (.004)	.074*** (.018)
s_u^2	.582*** (.025)	.526*** (.118)	.535*** (.020)	.500*** (.026)	.466*** (.033)	.503*** (.194)
?	.874*** (.007)	.848*** (.023)	.853*** (.008)	.862*** (.008)	.858*** (.010)	.872*** (.018)
Log-likelihood	-5184.546	-5144.909	-5146.440	-5084.126	-5118.405	-5086.361

Notes: *t*-ratios are asymptotic. *** $p < .001$; ** $p < .05$; * $p < .1$