Cooperative R&D under Uncertainty with Free Entry

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Abstract

In the last few decades, the effects of cooperative R&D arrangements on innovation and welfare have played an important role in policy making. Both the US and the EU have legislation in place providing for lenient antitrust treatment of such cooperative agreements. The goal of this paper is to analyze the effects of cooperative R&D arrangements in a model with a stochastic R&D process and output spillovers. Our main innovation is to allow for free entry in both the R&D race and the product market. To determine the desirability of cooperation in R&D environments and optimal antitrust policy, we compare three different ways of organizing R&D activities: R&D competition, R&D cartels, and RJV cartels. In contrast with the literature, we assume that cooperative R&D arrangements do not have to include all of the firms in the industry. Hence, the cartel members potentially face competition from outsiders both in the R&D race and the product market. The assumption of output spillovers together with free entry raises the prospect of innovation causing exit initially and entry later on. Our main conclusions are as follows. R&D cartels are always unprofitable, regardless of their size and the level of spillovers. RJV cartels, on the other hand, may be profitable and their profitability depends on their size. In particular, small RJV cartels are likely to be profitable whereas large RJV cartels are likely to be unprofitable. The aggregate rate of innovation with an R&D or an RJV cartel is the same as under R&D competition. However, this does not imply that profitable RJV cartels are always welfare improving. Since sharing of R&D outcomes affects the equilibrium number of firms in the product market after the R&D race, the consumer welfare effects of RJV cartels are sensitive to the specification of consumer preferences. Finally, we show that there may be a case for subsidizing cooperative R&D because some unprofitable R&D cartels and RJV cartels are welfare improving. These results extend the existing results in the literature in significant ways since in the literature, cooperative R&D arrangements are generally found to be profitable and their welfare effects depend on the level of spillovers.

JEL classification: L1, L4, O3

Keywords: Cooperative R&D; Research joint ventures; Free entry; Uncertain R&D; Technology spillovers.

1 Introduction

In the last few decades, the effects of cooperative R&D arrangements on innovation and welfare have played an important role in policy making. The observation that R&D environments are frequently characterized by substantial spillovers and appropriability problems has resulted in the concern that competing firms may have too little incentives to invest in R&D.[1] As a result, both the US and Europe have legislation for lenient antitrust treatment of research joint ventures (RJVs).[2]

A consistent feature of the literature on the effects of cooperative R&D arrangements has been the assumption that both the product market and the R&D process are characterized by high barriers to entry.[3] It is desirable to modify this assumption and study the impact of free entry for a number of reasons. First, after an R&D process, when the research outcomes are available to only a few firms, their resulting competitive advantage is likely to result in product market exit. However, as spillovers occur and the research outcomes become available to more and more firms, this creates incentives for entry into the product market. Allowing for such product market exit and entry introduces new welfare and investment implications of cooperative R&D. Second, in many R&D intensive industries, it may not be realistic to assume that only a limited number of firms can participate in the R&D process. In fact, in the closely related patent race literature, it is common to assume free entry into the R&D process.[4] When firms cooperate, they affect the expected profits of their rivals and, hence, may induce R&D race entry or exit. Thus, allowing for free entry into the R&D

[1] The discussion of the adverse effects of R&D spillovers and appropriability problems dates back to Arrow (1962). Empirically, Mansfield et al. (1981) show that 60% of the patented innovations in their sample were imitated within 4 years. Mansfield (1985) shows that information concerning development decisions leak out to rival firms within about 12 to 18 months.

[2] In the US, the National Cooperative Research and Production Act (NCRPA) of 1993 provides that research and production joint ventures be subject to a 'rule of reason' analysis instead of a per-se prohibition in antitrust litigation. In the EU, the Commission Regulation (EC) No 2659/2000 (the EU Regulation) provides for a block exemption from antitrust laws for RJVs, provided that they satisfy certain market share restrictions and allow all joint venture participants to access the outcomes of the research.

[3] For the existing theoretical literature on research joint ventures, see, for example, Kamien et al. (1992), Motta (1992), Suzumara (1992), Choi (1993), Vonortas (1994), Ziss (1994), Poyago-Theotoky (1995), Leahy and Neary (1997), Salant and Shaffer (1998), Martin (2002), Amir and Wooders (1999 and 2000), Amir (2000), Kamien and Zang (2000), and Anbarci et al. (2002). See De Bondt (1996) for an excellent survey.

[4] See, for example, Loury (1979), Lee and Wilde (1980), Reinganum (1985), and Denicolo (2000).

race introduces a new set of strategic implications of cooperative R&D.

The goal of this paper is, therefore, to analyze the effects of cooperative R&D arrangements in a model which allows for free entry into both the product market and the R&D process. Two other important assumptions which distinguish our model from the rest of the literature are the following. First, as in Miyagiwa and Ohno (2002), we model R&D as a stochastic process. This assumption is in contrast with the rest of the literature, where, following d'Aspremont and Jacquemin (1988), it is common to model R&D as a deterministic process.[5] Second, in accordance with industrial practice, we assume that cooperative R&D arrangements do not have to include all of the firms in the industry.[6] In comparison, it is generally assumed in the literature that cooperative R&D arrangements involve all of the firms in the industry.[7] By assuming that cooperative ventures do not face any competition from outsider participants in the R&D process, the existing literature fails to capture important strategic interactions generated by cooperative R&D arrangements.

In the set-up we consider, an R&D race is followed by oligopolistic product market competition. We assume that the winner of the R&D race has exclusive access to a quality-enhancing or cost-reducing innovation for a limited period of time, after which the innovation spills over to all of the firms in the industry. The duration of the exclusive access represents the speed of outcome spillovers. It is affected by the effectiveness of patent and/or trade secret protection.

Following the literature, we compare the following cooperative R&D arrangements with the benchmark case of R&D competition, where firms choose their R&D intensities independently. R&D cartel is the case where a fixed number of firms set their R&D investment levels to maximize their joint profits, but they do not share their R&D results. In case of an RJV cartel, a fixed number of firms choose their investment levels to maximize their joint profits and share their R&D results. That is, all participants get immediate access to the innovation as soon as any one of the participants wins the R&D race. Firms do

[5] In addition to Miyagiwa and Ohno (2002), three other papers that model R&D as a risky activity are Choi (1993), Martin (2002), and Hauenschild (2003).

[6] See, for example, Combs (1986).

[7] Kamien and Zang (1993) is a notable exception.

not cooperate in the product market in either of these arrangements. In both cases, we first focus on the case where the size of the cartel is such that some outsiders choose to participate in the R&D race. We later relax this assumption.

Our main findings are the following. We show that R&D cartels are always unprofitable irrespective of the speed of spillovers. Moreover, the per-firm investment level of the R&D cartel is lower than the per-firm investment level under R&D competition. Despite this, due to free entry in the R&D race, R&D cartels do not affect the aggregate rate of innovation whenever there are outsiders in the R&D race. In this case, however, they are still welfare reducing since they are unprofitable. If there are no outsiders in the R&D race, the aggregate rate of innovation is higher than under R&D competition, which implies that R&D cartels may be welfare-improving.

These results contrast with those in the literature. The most related paper to ours is Miyagiwa and Ohno (2002). They show in a model of stochastic R&D with barriers to entry that R&D cartels are always profitable, have a higher investment rate than the individual firms do under R&D competition, and increase the aggregate rate of innovation when outcome spillovers are sufficiently large. Their results are consistent with the rest of the literature, including papers which model input rather than outcome spillovers.[8]

Our study of RJV cartels reveals that their performance depends on their size. Small RJV cartels are more likely to be profitable and have higher per-firm investment levels than R&D competition while large RJV cartels are more likely to be unprofitable and have lower per-firm investment levels than R&D competition. As in the case of R&D cartels, the aggregate arrival rate of innovation is the same as under R&D competition if there are outsiders in the R&D race. If there are no outsiders, the aggregate rate of innovation is higher. However, in either case, welfare may still be lower with an RJV cartel because there may be fewer firms in the market when the RJV cartel develops the new technology.

These results are significant because in the literature where R&D cooperation is modelled assuming a deterministic R&D process and barriers to entry, RJV cartels are always more

[8] See, for example, Kamien et al. (1992).

profitable than and welfare superior to R&D competition. Moreover, they are most profitable when they include all of the firms in the industry.[9] These results from the literature do not explain the facts that firms choose to conduct many R&D projects non-cooperatively and that RJVs often do not include all of the firms in an industry. Miyagiwa and Ohno (2002) also qualify the conclusions of the deterministic R&D literature by showing that with uncertain R&D, the profitability and welfare implications of RJV cartels depend on the level of spillovers and the effect of sharing on industry profits.[10] In comparison, we show that when markets are characterized by free entry, the key variable for RJV cartel performance is its size.

Our analysis leads to the following policy conclusions. There may be a case for subsidizing cooperative R&D arrangements since they may be welfare improving even if they are not profitable. This conclusion is almost unique in the literature and contrasts with Leahy and Neary's (1997) conclusion that 'policy intervention to encourage cooperation is likely to be redundant whether or not it is desirable.'[11] In cases when cooperative R&D arrangements are profitable, their impact on welfare depends on consumers' preferences. This means that the optimal antitrust treatment of cooperative R&D may be different for different industries and a detailed analysis of demand may be required to determine the appropriate policy approach.[12]

The paper proceeds as follows. In Section 2, we present the details of the model. Section 3 presents the product market payoffs which we use in Sections 4, 5, and 6 in the analysis of R&D competition, R&D cartels, and RJV cartels respectively. Section 7 explores the welfare and policy implications of the cooperative R&D arrangements we consider. In Section 8, we extend the analysis by considering the effects of cooperative R&D arrangements when the cooperating firms face no competition from outsider participants in the R&D race in

[9] See Kamien and Zang (1993).

[10] Choi (1993) also finds that RJV cartels may be unprofitable because he assumes that sharing decreases product market payoffs.

[11] Choi (1993) also argues that it may be desirable to encourage cooperation, but for different reasons. In his model, cooperation always lowers aggregate industry profits and, hence, the social incentives for cooperation exceed the private incentives.

[12] We present in Erkal and Piccinin (2007) a class of models of oligopolistic competition with free entry where profitable cooperative R&D arrangements are always welfare improving.

equilibrium. We conclude and make suggestions for future research in Section 9.

2 The Model

Consider a continuous-time model where symmetric firms compete both in a product market and an R&D race for a new technology. There is free entry and exit in both the product market and the R&D race. We assume that the product market is in a long-run equilibrium when an opportunity for a new technology arises. Firms can compete either individually or jointly to be the first to develop the new technology, which may be either a quality-enhancing or a cost-reducing innovation.

We model research activity using a Poisson discovery process. Each firm operates an independent research facility. We allow both incumbent firms and new firms to participate in the R&D race. All participants must incur a one-time entry cost of S to enter the race. The entry cost represents the race-specific fixed-cost expenditure. Firms share a common discount rate r. Following Lee and Wilde (1980), we assume that the participants in the R&D race choose an R&D investment x and incur a flow cost x per unit of time. Investment provides a stochastic time of success that is exponentially distributed with hazard rate $h(x)$. We assume that $h'(x) > 0$, $h''(x) < 0$, and $h(0) = 0$. $\lim_{x \to 0} h'(x)$ is sufficiently large to guarantee an interior equilibrium and $\lim_{x \to \infty} h'(x) = 0$.

Following the literature, we consider the following three scenarios. Under R&D competition, the firms conduct research independently. They make their R&D and production decisions to maximize their individual payoffs. With an R&D cartel, a set $\mathbf{C} = \{1, ..., C\}$ of firms, which are exogenously designated to be part of the cartel, choose their R&D investments to maximize their joint profits. The cartel members do not share their research outcomes and compete in the product market. With an RJV cartel, a set $\mathbf{J} = \{1, ..., J\}$ of firms, which are exogenously designated to be part of the cartel, choose their R&D investments to maximize their joint profits *and* all participants in the cartel acquire the new technology when and if one of the cartel's members wins the R&D race. After sharing the new technology, the firms compete in the product market.

We let $\mathbf{R} = \{1, ..., R\}$ denote the set of all firms which choose to participate in the R&D race. We assume that if a cooperative R&D arrangement exists, R is strictly greater than the number of firms which cooperate. In other words, we assume that some outsiders always find it profitable to compete in the R&D race.[13]

We assume that the winner of the R&D race has exclusive rights to the use of the new innovation for a duration of T, after which time all firms immediately gain free access to the new technology. As stated in Miyagiwa and Ohno (2002), T can be interpreted as the speed of outcome spillovers. It is likely to be affected by the length and breadth of patent protection as well as the technological ease of reverse engineering.

In the product market, we assume that all firms incur fixed costs of production as long as they continue to produce. In other words, the fixed costs of production are an ongoing expense, not a one-off commitment. Payments on a renewable lease, utility fees, and head office costs are examples of these types of fixed costs. Since we assume that firms do not have sunk costs, they can therefore respond to new technological environments by entering and exiting without the complications that sunk costs create.

We make the following assumptions regarding the product market payoffs. We let N_t stand for the number of firms active in the product market at time t. Firms can choose to be active or inactive in the product market at every instant in the game. During the race, there continues to be free entry into the product market. All firms in the product market earn flow profits $\pi^o(N_t)$, where the superscript o stands for the old technology. After the R&D race ends but before the innovation spills over to the other firms in the industry, $\pi^w(W, N_t - W)$ and $\pi^f(W, N_t - W)$ denote the flow profits, net of fixed costs, earned by a firm with the new technology and a firm with the old technology, respectively. We let W stand for the number of winners at the end of the R&D race and $N_t - W$ stand for the number of active firms with the old technology in the market. Hence, if a single firm wins the race, that firm earns $\pi^w(1, N_t - 1)$ while if an RJV cartel wins the race, each member of the RJV cartel earns $\pi^w(J, N_t - J)$. We assume

[13] We relax this assumption in Section 8.

Assumption 1 $\pi^w(W, N_t - W) > \pi^f(W, N_t - W)$.

Hence, the firms with the new technology earn more than the firms with the old technology. Such an assumption holds in standard Cournot and differentiated good Bertrand models. After T periods, the innovation spills over and the firms' flow profits become $\pi^n(N_t)$, where the superscript n stands for the new technology, if they are active and zero otherwise.

We assume that all profit functions are decreasing in the level of competition, represented in the following two assumptions by the number of firms with the new technology and the total number of firms in the market, respectively.

Assumption 2 $\pi^w(W, N_t - W)$ and $\pi^f(W, N_t - W)$ are decreasing in W.

Assumption 3 All product market payoffs are decreasing in N_t.

To summarize, the timing of the game is as follows.

Phase 1: The product market is in a long-run equilibrium. An opportunity for the development of a new technology arises.

Phase 2: The cooperative R&D structure is determined exogenously.

Phase 3: All firms, whether active in the product market or not, decide whether to incur a fixed cost S to enter the R&D race. Those firms which are active in the product market earn flow profits $\pi^o(N_t)$.

Phase 4: The R&D race ends as soon as one of the firms develops the new technology. After a firm wins the R&D race but before spillovers occur, all firms with the new technology earn a flow payoff of $\pi^w(W, N_t - W)$ while all other active firms earn $\pi^f(W, N_t - W)$ for duration T.

Phase 5: After a duration of T, all firms gain access to the new technology and earn flow payoffs of $\pi^n(N_t)$ if they produce.

We solve for the subgame perfect equilibrium of this game using backward induction.

3 Product Market Competition

To determine the payoffs to the winners and losers of the R&D race, we start the analysis by considering the product market competition after the R&D race has ended. Note that the equilibrium number of firms in the product market changes only when there is a change in the technological environment. After the innovation spills over to all of the firms in the industry, entry occurs until $\pi^n(N_t) = 0$ and the firms earn zero profits thereafter.

There will be exit from the product market immediately after the R&D race ends since the presence of a firm with a better technology makes it unprofitable for some of the firms that were in the market before the race came to an end to continue to produce. The remaining firms continue to produce using the old technology. Since exit happens instantaneously, the remaining firms in the market immediately start to earn either $\pi^w(W, N_t - W)$ or $\pi^f(W, N_t - W)$. Hence, after the R&D race ends but before spillovers take place, N_t will either be equal to W or be determined by $\pi^f(W, N_t - W) = 0$.

4 R&D Competition

In this section we consider the benchmark case where the firms conduct R&D independently. We show that there exists a free entry equilibrium to the R&D competition game, and characterize the investment choices and the number of firms in this equilibrium. With a Poisson discovery process, the probability that there has not been a discovery until time t is given by $\exp\left[-\sum_{i\in\mathbf{R}} h(x_i)\, t\right]$. Conditional on this probability, each participant earns a flow profit of $(\pi^o(N_t) - x_i)\, dt$ during the interval dt if they are active in the product market and $-x_i dt$ otherwise. Since the product market is in a long-run equilibrium at the beginning of the game and the decision to be active in the product market is independent of the decision to be active in the R&D race, the number of firms in the market, N_t, does not change until the race ends and is given by $\pi^o(N_t) = 0$.

If firm i innovates during the interval dt, its earnings for the period T are

$$\int_0^T e^{-rt}\pi^w\left(1, N_t - 1\right)dt. \tag{1}$$

As explained in Section 3, N_t does not change until the end of the period T. Hence, we can re-write (1) as

$$\frac{L}{r} = \frac{\left(1 - e^{-rT}\right)\pi^w\left(1, N^R - 1\right)}{r}, \tag{2}$$

where N^R denotes the equilibrium number of firms in the product market during this time period.

Finally, if another firm innovates during the interval dt, firm i's profit is equal to zero due to the assumption of free entry and exit. Even after the spillovers occur and firm i gains access to the new technology, since entry will take place until the product market profits are driven to zero, firm i will continue to make zero.

We can now write down the present discounted value of the sum of firm i's expected profits over time as

$$\pi_i = \int_0^\infty e^{-\Sigma h(x_i)t}e^{-rt}\left[h\left(x_i\right)\frac{L}{r} - x_i\right]dt - S = \frac{h\left(x_i\right)\frac{L}{r} - x_i}{r + h\left(x_i\right) + \alpha_i} - S, \tag{3}$$

where $\alpha_i = \sum\limits_{j\neq i} h\left(x_j\right)$ stands for the aggregate hazard rate of the rival firms.

Given the memoriless nature of the Poisson process, firm i takes α_i as given and chooses x_i to maximize this payoff function at every point in time during the R&D race. The first order condition is

$$h'\left(x_i\right)\left[L + x_i + \frac{L}{r}\alpha_i\right] - \left[r + h\left(x_i\right) + \alpha_i\right] = 0. \tag{4}$$

The second order condition is

$$h''\left(x_i\right)\left[L + x_i + \frac{L}{r}\alpha_i\right] < 0,$$

which is always satisfied because of the concavity assumption on $h\left(x_i\right)$. Hence, the first order condition implicitly defines the optimal choice of x_i as a function of the rival firms' investment choices.

Since all of the firms are symmetric, we look for a symmetric equilibrium. The equilibrium individual investment levels and number of firms can be determined by solving the first order condition of a generic firm and the zero profit condition simultaneously. Taking R as given, we let $\overline{x}$ represent the symmetric solution to the first order conditions. It satisfies the following equation.

$$\overline{x} = \widehat{x}\left(\left(R-1\right)h\left(\overline{x}\right)\right), \tag{5}$$

where $\widehat{x}$ stands for the best response function. The corresponding per-firm profit level for a given value of R is

$$\overline{\pi} = \frac{h\left(\overline{x}\right)\frac{L}{r} - \overline{x}}{r + Rh\left(\overline{x}\right)} - S. \tag{6}$$

To determine the slope of the best response function of firm i, we can use the implicit function theorem. We have

$$\frac{\partial \widehat{x}\left(\alpha_i\right)}{\partial x_j} = -\frac{h'\left(x_j\right)\left[h'\left(\widehat{x}\right)\frac{L}{r} - 1\right]}{h''\left(\widehat{x}\right)\left[L + \widehat{x} + \frac{L}{r}\alpha_i\right]}. \tag{7}$$

Note that the first order condition given in (4) can be re-written as

$$\left[h'\left(x_i\right)\frac{L}{r} - 1\right]\left[r + h\left(x_i\right) + \alpha_i\right] - h'\left(x_i\right)\left[h\left(x_i\right)\frac{L}{r} - x_i\right] = 0.$$

From (3), we have $\pi_i + S = \frac{\left[h'(x_i)\frac{L}{r} - 1\right]}{h'(x_i)}$. Hence, we can re-write (7) as

$$\frac{\partial \widehat{x}\left(\alpha_i\right)}{\partial x_j} = -\frac{h'\left(x_j\right)h'\left(\widehat{x}\right)\left(\pi_i + S\right)}{h''\left(\widehat{x}\right)\left[L + \widehat{x} + \frac{L}{r}\alpha_i\right]} \tag{8}$$

which is > 0 since $h''\left(x_i\right) < 0$. Hence, the investment choices of the firms are strategic complements.

To show that there exists a free entry equilibrium, we need to show that profits are decreasing in R. The first step in this analysis is to show that $\overline{x}$ is increasing in R. Using (5) we get

$$\frac{\partial \overline{x}}{\partial R} = \frac{\partial \widehat{x}\left(\left(R-1\right)h\left(\overline{x}\right)\right)}{\partial R} = \frac{\frac{\partial \widehat{x}}{\partial (R-1)h(\overline{x})}h\left(\overline{x}\right)}{1 - \frac{\partial \widehat{x}}{\partial (R-1)h(\overline{x})}\left(R-1\right)h'\left(\overline{x}\right)}.$$

The numerator of this expression is positive because the investment decisions of the firms are strategic complements. Following Lee and Wilde (1980), we define the expression in the denominator as a stability condition. Hence, it is also positive and $\frac{\partial \overline{x}}{\partial R} > 0$. Second, since $\alpha_i = \overline{\alpha} = (R-1)\, h(\overline{x})$ for a given value of R, this implies that $\frac{\partial \overline{\alpha}}{\partial R} > 0$. Finally, we can show that the present discounted value of expected profits, given in (6), is decreasing in R using the following lemma.

Lemma 1 *The expected payoff of a profit maximizing firm i that is active in the R&D race decreases monotonically with the value of α_i.*

Proof. Applying the envelope theorem to the maximum value of (3) gives

$$\frac{-\left[h\left(\widehat{x}\left(\alpha_i\right)\right)\frac{L}{r} - \widehat{x}\left(\alpha_i\right)\right]}{\left[r + h\left(\widehat{x}\left(\alpha_i\right)\right) + \alpha_i\right]^2} < 0.$$

■

Since Lemma 1 states that a profit maximizing firm's payoff is decreasing in α_i and we have shown that for any given R, $\overline{\alpha}$ is increasing in R, we can conclude that $\overline{\pi}(R)$ is decreasing in R. This means that there exists a free entry equilibrium to the game in which the equilibrium number of firms under R&D competition, R^*, is determined by $\pi^* = \overline{\pi}(R^*) = 0$ and each of these R^* firms invests $x^* = \overline{x}(R^*)$.

5 R&D Cartel

In this section, we compare the case of R&D competition with the case where a group of C firms form an R&D cartel. The cartel members participate in the R&D race by choosing their investment levels to maximize their joint payoffs, but they do not share their research outcomes. Hence, as in the case of R&D competition, each firm i in the cartel earns $\frac{L}{r}$ only in the case when it wins the R&D race. We assume that due to free entry in the R&D race, the cartel participants still face competition from outsider participants in the R&D race.

The joint payoffs of the cartel participants are given by

$$\sum_{i\in\mathbf{C}}\left(\frac{h\left(x_i\right)\frac{L}{r} - x_i}{r + \sum\limits_{j\in\mathbf{C}} h\left(x_j\right) + \sum\limits_{k\notin\mathbf{C}} h\left(x_k\right)} - S\right), \tag{9}$$

where $\mathbf{C}$ is defined as the set of firms participating in the R&D cartel, L is as defined in (2), and the last term in the denominator, $\sum_{k \notin \mathbf{C}} h(x_k)$, stands for the sum of the hazard rates of the outsider participants in the R&D race. The outsider participants maximize the payoff function given in (3). Solving the first order conditions of the cartel members and the outsider participants together with the zero profit condition on outsiders' payoff function yields the equilibrium investment amounts and number of firms.

We first show that as in the case of R&D competition, there exists a free entry equilibrium with an R&D cartel. To show this, we define $\overline{x}^C$ and $\overline{x}^O$ as the investment levels which satisfy the first order conditions of the cartel participants and the outsider firms, respectively, for a given number of cartel participants, C, and outsiders, $O = R - C$. As in the case of R&D competition, we show in the following lemma that $\overline{x}^C$ and $\overline{x}^O$ are both increasing in R by invoking a stability condition.

Lemma 2 *$\overline{x}^C$ and $\overline{x}^O$ are both increasing in R.*

Proof. See Section 1 in the Appendix. ■

Lemma 2 implies that for any given outsider firm, $\overline{\alpha}_i = Ch\left(\overline{x}^C\right) + (R - C - 1)\, h\left(\overline{x}^O\right)$ must be increasing in R. Since by Lemma 1 the maximized profits of an outsider firm are decreasing in α_i, we can conclude that there exists a free entry equilibrium where R^C denotes the number of participants in the R&D race and all outsider participants earn zero profits.

We let x^C stand for the optimal symmetric investment level of each cartel participant in a free entry equilibrium. It is implicitly defined by the following first order condition, where α^C denotes the equilibrium aggregate hazard rate of the outsider participants in the R&D race.

$$h'\left(x^C\right)\left[L + Cx^C + \frac{L}{r}\alpha^C\right] - \left[r + Ch\left(x^C\right) + \alpha^C\right] = 0.$$

We would like to determine the profitability of R&D cartels and their impact on innovation. We start by comparing the per-firm investment level under R&D competition with the per-firm investment level in an R&D race with an R&D cartel. The following proposition establishes that while the cartel participants reduce their per-firm investment level, the outsider participants invest the same amount as the firms do under R&D competition.

Proposition 1 *In equilibrium, the per-firm investment level of the R&D cartel participants,* x^C*, is lower than the per-firm investment level under R&D competition,* x^**. The investment level of outsider firms in the R&D race with an R&D cartel,* x^O*, is equal to* x^**.*

Proof. See Section 2 in the Appendix. ■

With free entry in the product market, R&D investments always confer a net negative externality on rivals. Outcome spillovers do not provide any benefit to the firms that lose the R&D race because any future rents that could be earned by using the new technology are dissipated through entry into the product market. This transforms the R&D race into a winner-takes-all game. Since the R&D cartel members internalize the negative externality they impose on each other, they invest less than the outsider participants in the R&D race.

In the proof of Proposition 1, we use Lemma 1 to show that all outsider firms invest the same amount in the presence of an R&D cartel as they do under R&D competition. This is because in both cases the outsider firms earn zero profits in equilibrium and, therefore, face the same profit maximization problem and choose the same solution.

Comparing these results with those of Miyagiwa and Ohno (2002) reveals the importance of the assumption of free entry. If there are barriers to entry in the product market, the losers of the R&D race still get a chance to benefit from the innovation after spillovers happen. As they show, for sufficiently small values of T (i.e., for sufficiently rapid spillovers), this positive spillover effect outweighs the negative competitive effect and, thus, R&D investments confer a net positive externality on the rival firms. Hence, Miyagiwa and Ohno (2002) find that for sufficiently small T values, when firms form an R&D cartel that allows them to internalize these positive externalities, they end up increasing their investment levels above the investment level under R&D competition.[14]

[14] This result is in line with the results of the other papers in the literature that model R&D as a deterministic process. If the rate of spillovers is sufficiently high, the positive externality they generate outweighs the negative externality generated by competition. See, for example, d'Aspremont and Jacquemin (1988) and Kamien et al. (1992).

Ultimately, what is important is the impact of the R&D cartel on the aggregate arrival rate of innovation. Due to the assumption of free entry, one cannot readily use the results on the changes in the individual investment levels to determine the impact of R&D cartels on the aggregate rate of innovation. Since there are outsider firms in the R&D race, we can write the aggregate arrival rate of innovation as equal to $\alpha_i + h(x_i)$ for any outsider firm i. Since the outsider firms earn zero profits both with an R&D cartel and under R&D competition, Lemma 1 implies that α_i is the same in both cases. From Lemma 1 we know that $x_i = x^*$ in both cases also. Hence, the aggregate rate of innovation is the same in both cases.

This discussion also allows us to conclude that the total number of firms participating in the R&D race is higher with an R&D cartel than under R&D competition. This is because the aggregate arrival rate of innovation is the same with an R&D cartel as under R&D competition and $x_C < x^O = x^*$.

We summarize this discussion in the following proposition.

Proposition 2 *In an R&D environment with an R&D cartel,*

(i) the aggregate arrival rate of innovation is the same as under R&D competition;

(ii) a higher number of firms participate in the R&D race than under R&D competition.

In the literature, R&D cartels decrease innovation for sufficiently low spillovers and increase it for sufficiently high spillovers. This follows immediately from the per-firm investment results discussed above since it is generally assumed that all firms participate in the R&D cartel. Our results do not depend on the level of spillovers because of free entry into the R&D race.

We finally evaluate the profitability of R&D cartels.

Proposition 3 *All R&D cartels are unprofitable.*

Proof. Each member of the R&D cartel and each outsider firm face $R^C - 1$ competitors in equilibrium. Of these, $R^C - 2$ competitors are the same for these two firms. However, an R&D cartel member's $R^C - 1$th competitor is an outsider firm investing x^* while the outsider firm's $R^C - 1$th competitor is an R&D cartel member investing $x^C < x^*$. Hence, the value of α_i faced by a member of the R&D cartel is higher than the value of α_i faced by an outsider firm. By Lemma 1, this implies that the outsider firm would earn a higher profit level than the R&D cartel member if both firms maximized their individual profits. However, the R&D cartel member earns even lower profits since it does not maximize its individual profits. ■

This result also contrasts with the results in the previous studies of R&D cartels, which consistently find that the joint profits of the firms within an R&D cartel are higher than their joint profits under R&D competition.[15] In our analysis, free entry into the R&D race makes otherwise profitable R&D cartels unprofitable. This is because the members of an R&D cartel earn less than the outsider participants in the R&D race because of a free rider effect.[16] The outsider firms benefit from the lower investment of the cartel members because it increases their probability of success. Since the outsiders earn zero, and the R&D cartel members earn less, the R&D cartels are unprofitable in equilibrium.[17]

6 RJV Cartel

We next consider the case where an exogenously-determined group of J firms participate in the R&D race by forming an RJV cartel. The firms make their R&D decisions jointly and gain immediate access to the new technology in the event that any one of them wins the R&D race.

The RJV cartel's expected payoff is

$$\sum_{i\in\mathbf{J}}\left(\frac{h(x_i)\frac{L^J}{r}+\sum\limits_{k\neq i,k\in\mathbf{J}}h(x_k)\frac{L^J}{r}-x_i}{r+h(x_i)+\sum\limits_{k\neq i,k\in\mathbf{J}}h(x_k)+\sum\limits_{l\notin\mathbf{J}}h(x_l)}-S\right), \tag{10}$$

[15] Again, see, for example, d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and Miyagiwa and Ohno (2002).

[16] A similar kind of free rider effect exists in the mergers literature. See Salant et al. (1983) and Deneckere and Davidson (1985).

[17] Erkal and Piccinin (2006) and Davidson and Mukherjee (2007) show that free entry in the product market makes otherwise profitable product market mergers unprofitable.

where the last term in the denominator stands for the sum of the hazard rates of the outsider participants in the R&D race and L^J is given by

$$L^J = \left(1 - e^{-rT}\right) \pi^w \left(J, N^J - J\right),$$

where N^J stands for the equilibrium number of firms in the product market during the period T. The flow profit the firms earn if they win the race, $\pi^w \left(J, N^J - J\right)$, depends on the number of participants in the RJV cartel because the winner shares the new technology with the rest of the cartel, which determines the number of firms in the product market with the new technology for the period T.

We first show that as in the case of R&D competition and R&D cartel, there exists a free entry equilibrium with an RJV cartel. To show this, we define $\overline{x}^J$ and $\overline{x}^O$ as the investment levels which satisfy the first order conditions of the cartel participants and the outsider firms, respectively, for a given number of cartel participants, J, and outsiders, $O = R - J$. We show in the following lemma that $\overline{x}^J$ and $\overline{x}^O$ are both increasing in R by invoking a stability condition analogous to the one in Section 5.

Lemma 3 *$\overline{x}^J$ and $\overline{x}^O$ are both increasing in R.*

Proof. See Section 3 in the Appendix. ■

Lemma 3 implies that for any given outsider firm, $\overline{\alpha}_i = Jh\left(\overline{x}^J\right) + (R - J - 1)\, h\left(\overline{x}^O\right)$ must be increasing in R. Since by Lemma 1 the maximized profits of an outsider firm are decreasing in α_i, we can conclude that there exists a free entry equilibrium where R^J denotes the number of participants in the R&D race and all outsider participants earn zero profits..

We let x^J stand for the optimal symmetric investment level of each cartel participant and α^J stand for the equilibrium aggregate hazard rate of the outsider participants in the R&D race. x^J is implicitly defined by the following first order condition.[18]

$$Jh'\left(x^J\right)\left[L^J + x^J + \frac{L^J}{r}\alpha^J\right] - \left[r + Jh\left(x^J\right) + \alpha^J\right] = 0.$$

[18] It is straightforward to verify that the second order condition holds because of the concavity assumption on $h\left(x_i\right)$.

The RJV cartel's equilibrium payoff level, Π^J, is equal to

$$\Pi^J = J\left(\frac{Jh\left(x^J\right)\frac{L^J}{r} - x^J}{r + Jh\left(x^J\right) + \alpha^J} - S\right). \tag{11}$$

To analyze the impact of RJV cartels, we start by comparing them to R&D cartels. The difference between an RJV cartel and an R&D cartel is the product market payoff the members receive when one of the cartel participants win the R&D race. Hence, as a first step, we explore how the equilibrium per-firm investment and profit levels of an RJV cartel change with L^J. While analyzing the impact of an increase in L^J, one has to take into account its effect on the entry and investment decisions of the outsider participants in the R&D race also. We have the following result.

Lemma 4 *The equilibrium per-firm investment and profit levels of an RJV cartel of size J are monotonically increasing in L^J.*

Proof. See Section 4 in the Appendix. ■

We next evaluate the performance of an RJV cartel for low and high values of L^J to be able to draw conclusions for the range of possible RJV cartel effects. The following lemma presents results for the cases when $L^J = \frac{L}{J}$ and $L^J = L$.

Lemma 5 *If $L^J = \frac{L}{J}$, in equilibrium the members of the RJV cartel invest less than x^*, the per firm investment level under R&D competition, and make a lower profit than they would under R&D competition. If $L^J = L$, in equilibrium the members of the RJV cartel invest more than x^* and make a higher profit than they would under R&D competition.*

Proof. See Section 5 in the Appendix. ■

In the comparison of the per-firm investment levels in an environment with an RJV cartel and in an environment with R&D competition, two effects play a role. First, while firms inflict negative externalities on each other under R&D competition, members of RJV cartels confer positive externalities on each other because they share their research outcomes. Joint profit maximization allows the cartel participants to internalize these positive externalities, which causes the per-firm investment level to increase. Second, the per-firm returns to winning when the firms are part of an RJV cartel differ from those under R&D competition because when a member of the RJV cartel wins the R&D race, all its members have access to the new technology. When $L^J = \frac{L}{J}$, the returns with an RJV cartel are lower than the returns under R&D competition, which are equal to L. This affects the per-firm investment level with an RJV cartel adversely. Lemma 5 implies that when $L^J = \frac{L}{J}$, this negative effect dominates the positive effect and, hence, the members of the RJV cartel invest less than they would under R&D competition. On the other hand, when $L^J = L$, the returns to winning are the same under both arrangements and the first effect causes the per-firm investment to be higher with an RJV cartel.

Considering RJV cartels which include all of the firms in an industry, Miyagiwa and Ohno (2002) show that if it is the case that the industry profits are at least as high when the RJV cartel has the new technology as they are when only one firm has the new technology, the members of the RJV cartel make higher profits than they would under R&D competition.[19] Lemma 5 implies that this is not the case with free entry. To see this, note that when $L^J = \frac{L}{J}$, the RJV cartel's total payoff when one of its members wins the race is equal to L, the payoff from winning the race for a single profit-maximizing firm under R&D competition. In other words, ex post industry profits are the same if the RJV cartel wins the race as they are if a single firm wins since the losing firms in the race make zero profits due to free entry. The per-firm ex-ante payoff level of the RJV cartel is lower in this case than the per-firm ex-ante payoff level under R&D competition. Lemma 4 and 5 imply that with free entry, the industry profits have to be strictly higher with an RJV cartel for the RJV cartel's per-firm profit level to be higher.

As far as RJV cartels and R&D cartels are concerned, we point out in the proof of Lemma

[19] Using a model of stochastic R&D process, Choi (1993) also finds that RJV cartels may be unprofitable because in his model sharing of R&D outcomes increases product market competition. Such an assumption is not required for our results. Indeed, in our framework, it is possible for sharing of R&D outcomes to increase industry profits since it can also reduce the total number of participants in the product market. RJV cartels may be unprofitable even in those cases when sharing increases industry profits because of free entry in the R&D race. In contrast with these papers which model R&D as a stochastic process, in papers which model R&D as a deterministic process, RJV cartels are always profitable and result in an increase in the level of investment because of spillovers and joint-profit maximization.

5 that when $L^J = \frac{L}{J}$, an RJV cartel and an R&D cartel of the same size would result in the same level of per-firm investment and would earn the same profits in equilibrium. Together with Lemma 4 this implies that for $L^J > \frac{L}{J}$, firms make higher investments and higher profits in an RJV cartel than in an R&D cartel. This result is similar to the result in Miyagiwa and Ohno (2002). This is because the analyses of RJV cartels and R&D cartels only differ in the ex-post rewards for winning the race and free entry does not alter the basic insight that per-firm investment and payoff should be increasing in these rewards.

Using Lemma 4 and 5, we can establish the existence of two critical values, $\widehat{L}^J(J)$ and $\widetilde{L}^J(J)$, such that the per-firm profit and investment levels will be higher with an RJV cartel than under R&D competition if $L^J > \widehat{L}^J(J)$ and $L^J > \widetilde{L}^J(J)$ respectively. In the proof of Proposition 4, we show that $\widehat{L}^J(J) > \widetilde{L}^J(J)$. The following proposition presents a characterization of the performance of R&D environments with RJV cartels based on L^J using these critical values.

Proposition 4 *For values of J such that $L^J > \widetilde{L}^J(J) \in \left(\widehat{L}^J(J), L\right)$, the members of an RJV cartel invest higher amounts and make higher profits than they would under R&D competition. For values of J such that $L^J \in \left[\widehat{L}^J(J), \widetilde{L}^J(J)\right]$, the members of an RJV cartel invest lower amounts and make higher profits than they would under R&D competition. For values of J such that $L^J < \widehat{L}^J(J) \in \left(\frac{L}{J}, L\right)$, the members of an RJV cartel invest lower amounts and make lower profits than they would under R&D competition.*

Proof. See Section 6 in the Appendix. ■

Proposition 4 implies that if the per-firm investment level is higher with an RJV cartel, it must be the case that the per-firm profit level is also higher. The reason that the firms may be making higher profits even if their investment levels are lower than they would be if they were not cooperating is that they benefit from each other's investment levels. Being a member of the RJV cartel provides them with insurance in that they start to earn L^J as soon as any member of the cartel successfully develops the new technology. Hence, with lower individual investment amounts, they can still have higher individual expected payoff levels than they would under R&D competition.

Proposition 4 allows us to link RJV cartel size to RJV cartel performance if we impose a weak condition on the relationship between L^J and J. We do this in the following assumption.

Assumption 4 $\frac{\partial L^J(J)}{\partial J} \leqslant 0$ and $\lim_{J \to \infty} L^J(J) < 0$.

Assumption 4 states that the returns from winning the R&D race are weakly decreasing in the size of the RJV cartel. This assumption would be satisfied in many standard models of oligopolistic product markets with free entry.[20]

Given Assumption 4, the following corollary follows immediately from Proposition 4.

Corollary 1 *The members of a sufficiently small RJV cartel invest higher amounts and make higher profits than they would under R&D competition. The members of an intermediate-sized RJV cartel invest lower amounts and make higher profits than they would under R&D competition. The members of a sufficiently large RJV cartel invest lower amounts and make lower profits than they would under R&D competition.*

The per-firm payoff to winning the R&D race, L^J, depends on the RJV cartel's size, J. For sufficiently large RJV cartels such that $L^J < \widehat{L}^J(J)$, these payoffs are lower because the innovation is shared amongst more firms in the product market, which increases the product market competition. Corollary 1 compares RJV cartels of various size categories with the benchmark of R&D competition. It provides an explanation for why relatively small RJV cartels may form - these may be more profitable than larger ones. Indeed, some large RJV cartels may be unprofitable because they include too many firms.

This analysis makes an important contribution to the literature since studies of cooperative arrangements in R&D environments generally assume that all of the firms in the industry participate in the cooperative structure. Hence, they do not analyze the impact of RJV cartel size on performance. An exception is Kamien and Zang (1993). Using a model with barriers to entry and a deterministic R&D process, they find that if the firms in an

[20] In Erkal and Piccinin (2007), we present such a model where $L^J(J)$ is constant for all J for which there are product market outsiders and strictly decreasing in J thereafter.

industry form competing RJV cartels, the resulting investment level may be higher than if all of the firms were members of the same grand RJV cartel. This is because the competition effect can dominate the externalities effect. That is, although in environments with competing RJV cartels each RJV cartel benefits less from joint profit maximization because fewer firms benefit from each member's investment amount, the firms may still invest more in such environments because of the competitive pressure they exert on each other. This is similar to what is driving the result in our paper.

However, in Kamien and Zang (1993), firms always make higher profits with a grand RJV than with competing RJVs. In contrast, we find that smaller RJV cartels may be more profitable than larger RJV cartels. This is essentially because of the competition that the outsiders in the R&D race provide. Whereas having a larger RJV in Kamien and Zang's (1993) framework reduces competition in R&D, it does not necessarily have that effect in our model due to the free entry and exit of outsider firms.

Finally, we turn our attention to the impact of RJV cartels on the aggregate arrival rate of innovation. Since the analysis is identical to the analysis in the case of R&D cartels, which precedes Proposition 2, we do not repeat it here. We get the following result.

Proposition 5 *The aggregate arrival rate of innovation with an RJV cartel is the same as under R&D competition.*

In the literature on RJV cartels with barriers to entry, since RJV cartels are assumed to include all of firms in the industry, the results on the aggregate arrival rate of innovation follow immediately from the results on the per-firm investment level with the RJV cartel. In the deterministic R&D literature, this means that RJV cartels always increase the aggregate arrival rate of innovation. In contrast, in Miyagiwa and Ohno (2002), the impact of RJV cartels on the aggregate arrival rate of innovation depends on the level of spillovers and the effect of sharing on industry profits. Our results indicate that with free entry, even if the per-firm investment level is different from the R&D competition level, the aggregate arrival rate of innovation is always the same.

7 Welfare and Policy Implications

We next turn our attention to the welfare and policy implications of the cooperative R&D arrangements that we have considered. We define welfare as the sum of consumer welfare and producer surplus. Since the firms make zero profits in equilibrium, welfare under R&D competition is equal to

$$W^* = \frac{R^* h(x^*) \frac{\Omega^1}{r} + \omega^o}{r + R^* h(x^*)},$$

where

$$\frac{\Omega^1}{r} = \frac{\left(1 - e^{-rT}\right) \omega^1 + e^{-rT} \omega^n}{r} \tag{12}$$

stands for the consumer welfare level after the race ends and one firm has the new technology for duration T. ω^o, ω^1, and ω^n stand for the flow consumer welfare when no firms, only one firm, and all firms have access to the new technology respectively.

Similarly, the equilibrium welfare expressions with an R&D and RJV cartel are

$$W^C = \frac{\left[Ch\left(x^C\right) + \left(R^C - C\right) h(x^*)\right] \frac{\Omega^1}{r} + \omega^o + C\left[h\left(x^C\right) \frac{L}{r} - x^C\right]}{r + Ch\left(x^C\right) + \left(R^C - C\right) h(x^*)} - C \cdot S$$

and

$$W^J = \frac{Jh\left(x^J\right) \frac{\Omega^J}{r} + \left(R^J - J\right) h(x^*) \frac{\Omega^1}{r} + \omega^o + J\left[h\left(x^J\right) \frac{JL^J}{r} - x^J\right]}{r + Jh\left(x^J\right) + \left(R^J - J\right) h(x^*)} - J \cdot S$$

respectively. Defining ω^J as the flow consumer welfare when J firms have the new technology,

$$\frac{\Omega^J}{r} = \frac{\left(1 - e^{-rT}\right) \omega^J + e^{-rT} \omega^n}{r} \tag{13}$$

stands for the consumer welfare level after the race ends and J firms have the new technology for duration T.

These expressions show that since we evaluate welfare from an ex ante perspective, the aggregate arrival rate of innovation determines how rapidly consumers start to benefit from the new technology and firms start to make profits from it. Although Propositions 2 and 5 state that the aggregate rate of innovation remains unchanged with R&D and RJV cartels, we show in the following discussion that they may still affect welfare adversely.

7.1 R&D Cartels

Since the innovation arrives at the same time in expectation whether or not there is an R&D cartel, we have

$$W^C - W^* = \frac{C\left[h\left(x^C\right)\frac{L}{r} - x^C\right]}{r + Ch\left(x^C\right) + \left(R^C - C\right)h\left(x^*\right)} - C \cdot S. \tag{14}$$

That is, the only difference between the welfare level with an R&D cartel and the welfare level under R&D competition is the expected profits of the R&D cartel members themselves. This is because in both cases there is only one firm with the new technology in the market for the duration T after the R&D race ends. This implies that consumer welfare is the same in expectation whether an R&D cartel is formed or not.

Since we know from Proposition 3 that R&D cartels earn negative profits, (14) implies that they must be welfare decreasing. Hence, our analysis implies that in industries with free entry, since R&D cartels would never arise, antitrust policy towards them is irrelevant. Moreover, since they always decrease welfare, it is not desirable to subsidize R&D cartels in order to make them profitable if there are outsider participants in the R&D race.

7.2 RJV Cartels

As in the case of R&D cartels, since $Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^*\right) = R^* h\left(x^*\right)$, we have

$$W^J - W^* = \frac{Jh\left(x^J\right)\frac{\Omega^J}{r} + \left(R^J - J - R^*\right)h\left(x^*\right)\frac{\Omega^1}{r}}{r + Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^*\right)} + J\left[\frac{h\left(x^J\right)\frac{JL^J}{r} - x^J}{r + Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^*\right)} - S\right].$$

That is, the only difference between consumer welfare with an RJV cartel and under R&D competition is that when an RJV cartel wins the race but before spillovers occur, there are J firms with the new technology rather than only one. Hence, from (12) and (13) we can conclude that any profitable RJV cartel is also welfare improving if $\omega^J \geqslant \omega^1$. While one may expect consumer welfare to be increasing in the number of firms with access to the new technology, this may not always be the case because increasing the number of firms with the new technology causes greater exit of firms with the old technology. Therefore, there may be fewer firms active in the product market when an RJV cartel wins the race than when a single firm does. In general, the net effect on consumer welfare of an RJV cartel can go either way depending on consumers' preferences and, hence, profitable RJV cartels may present a welfare trade-off between lower expected consumer welfare and higher expected profits.

This analysis implies that in industries with free entry, antitrust policy should pay careful attention to consumers' preferences and may, therefore, differ between industries.[21] This contrasts with the policy prescriptions in the literature with barriers to entry and deterministic R&D, where RJV cartels are always found to be welfare improving and, therefore, should be allowed. Miyagiwa and Ohno (2002) reach a more cautious conclusion. They find that it is both privately and socially optimal to form an RJV cartel if spillovers are fast and industry profits from sharing exceed those without sharing.[22] In our model, however, these two conditions together are neither necessary nor sufficient for the social and private incentives for RJV cartels to coincide.

The analysis also implies that there may be a case for subsiding unprofitable RJV cartels when they are welfare improving. This conclusion is a major departure from the results in the literature with barriers to entry and deterministic R&D, where Leahy and Neary (1997) conclude, for instance, that 'policy intervention to encourage cooperation is likely to be redundant whether or not it is desirable.' In the case of uncertain R&D, Choi (1993) and Miyagiwa and Ohno (2002) do find room for subsidizing RJV cartels.[23] In particular, Choi (1993) concludes that the social incentives to form RJVs always exceed the private incentives. However, Choi's (1993) results depend upon the assumption that sharing results increases product market competition. This assumption is not necessary for our results.

[21] In Erkal and Piccinin (2007) we explore a class of models of oligopolistic product market competition with free entry and show that consumer welfare is always at least as high when J firms have the new technology as when a single firm has the new technology. This class includes as special cases homogeneous product Cournot competition, differentiated products Cournot competition with separable utility, and differentiated products Bertrand competition with separable indirect utility. In these cases, we can therefore conclude that all profitable and some unprofitable RJV cartels are welfare improving.

[22] In all other circumstances, they cannot guarantee that the private and social incentives to cooperate will coincide.

[23] Combs (1992) also argues that RJVs may be unprofitable. This is because in her model, industry profits are always lower if the results are shared than when only one firm has the new technology.

8 Cooperative R&D without R&D race outsiders

In the analysis above we have maintained the assumption that some outsiders always find it profitable to enter the R&D race in equilibrium. In this section, we provide some additional insights about cooperative R&D with free entry for the case where no outsiders choose to enter the race. We do this to address the potential concern that cooperation between firms in the R&D race may induce the exit of outsiders and, thus, reduce competition in the R&D race. Our results in this section show that the prospect of cooperative R&D having this effect is no cause for concern.

The main difference in results from the case with outsiders concerns the aggregate rate of innovation. Surprisingly, we show in the following proposition that without outsiders, the aggregate rate of innovation must be higher with either an R&D or an RJV cartel than under R&D competition.

Proposition 6 *If there are no outsiders in equilibrium, the aggregate rate of innovation with an R&D cartel or an RJV cartel must be higher than under R&D competition.*

Proof. We present the proof for the case of an R&D cartel only since the case of an RJV cartel is identical. Since the outsider participants always invest x^* in a free entry equilibrium and the aggregate rate of innovation is the same both with an R&D cartel and under R&D competition, the number of outsider participants in equilibrium is defined by

$$R^* h(x^*) = Oh(x^*) + Ch\left(x^C\right),$$

where O stands for the number of outsider participants. This implies that whenever $Ch\left(x^C\right) < R^* h(x^*)$, the equilibrium number of outsider participants in the R&D race must be strictly positive. This means that if no outsiders find it profitable to participate in the R&D race, we must have $Ch\left(x^C\right) \geqslant R^* h(x^*)$ and the aggregate arrival rate of innovation must be higher with an R&D cartel than under R&D competition. ■

The reason for this result is that if outsiders find it unprofitable to enter the race, it must be because the cooperating firms have collectively invested enough to ensure any entry would be unprofitable.

We show in Appendix 7 that most of the other results from the previous analysis continue to hold without R&D race outsiders. In particular, we show that all R&D cartels are unprofitable and their per-firm investment is less than x^*. Moreover, there are critical values of L^J above which RJV cartels invest more per-firm than x^* and are profitable.

The only policy conclusion that is qualitatively different from those we reached in Section 7 is that it may be desirable to subsidize R&D cartels. This is because R&D cartels increase the aggregate rate of innovation and, therefore, increase consumer welfare in expectation. Hence, R&D cartels without R&D race outsiders present a welfare trade-off between lower profits and higher consumer welfare. To the best of our knowledge, the conclusion that subsidies for R&D cartels may be socially desirable is unique in the literature since they are always found to be profitable.

9 Conclusion

We have analyzed the effects of cooperative R&D in a model of free entry with stochastic R&D and an oligopolistic product market. In contrast with the results in the literature, we have shown that R&D cartels are always unprofitable and never affect the aggregate rate of innovation adversely in equilibrium. RJV cartels, on the other hand, can be profitable depending on their size. Similar to R&D cartels, they also never adversely affect the aggregate rate of innovation.

Our findings are important because they account for the effects of entry and exit in R&D environments which have been missing from the literature to date. In our framework, winners of the R&D race always induce exit in the product market when they develop the new technology. However, when the technology spills over to other firms, entry occurs in the product market. Because R&D cartels invest less per firm than the outsider participants in the R&D race, there are always more firms in an R&D race in the presence of an R&D cartel than under R&D competition. This explains why R&D cartels are unprofitable. RJV cartels, on the other hand, may cause there to be more or fewer firms in the R&D race and may cause more product market exit than a single firm does after winning the R&D race.

Both the standard approach of modelling cooperative R&D with barriers to entry and our approach of free entry can be understood as opposite ends of a spectrum. This paper offers some guidance as to how the existing literature's policy prescriptions may change as entry conditions vary along this continuum. Our results indicate that it may be desirable to subsidize R&D cartels in cases when there are no outsider participants in the R&D race. Such a policy conclusion does not find support in the existing literature which assumes barriers to entry because a consistent conclusion of this literature is that R&D cartels are always profitable. The results also imply that since sharing of R&D outcomes affects the equilibrium number of firms in the product market after the R&D race, the consumer welfare effects of RJV cartels are sensitive to the specification of consumers' preferences. Hence, antitrust policy towards RJV cartels should be sensitive to consumers' preferences. Moreover, there may be a case for subsidizing RJV cartels with or without outsider participants in the R&D race because some unprofitable RJV cartels are welfare improving.

Future research should consider the effects of input spillovers on cooperative research with free entry. The assumption of a Poisson discovery process used in this paper may not be appropriate for such a study because of the assumption that the research paths embarked upon by firms are independent. If one firm's research project benefits from the efforts of a rival firm, it would seem more reasonable to assume that their instantaneous probabilities of success should be correlated.

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Appendix

1 Proof of Lemma 2

$\overline{x}^C$ and $\overline{x}^O$ are defined by the following first order conditions:

$$\overline{G}^C \equiv h'\left(\overline{x}^C\right)\left[L + C\overline{x}^C + \frac{L}{r}\left(R-C\right)h\left(\overline{x}^O\right)\right] - \left[r + Ch\left(\overline{x}^C\right) + \left(R-C\right)h\left(\overline{x}^O\right)\right] = 0 \tag{A.1}$$

and

$$\overline{H}^C \equiv h'\left(\overline{x}^O\right)\left[L + \overline{x}^O + \frac{L}{r}\left[\left(R-C-1\right)h\left(\overline{x}^O\right) + Ch\left(\overline{x}^C\right)\right]\right] - \left[r + Ch\left(\overline{x}^C\right) + \left(R-C\right)h\left(\overline{x}^O\right)\right] = 0.$$

Totally differentiating these and applying Cramer's Rule gives

$$\frac{d\overline{x}^C}{dR} = \frac{-\frac{\partial\overline{G}^C}{\partial R}\frac{\partial\overline{H}^C}{\partial\overline{x}^O} + \frac{\partial\overline{G}^C}{\partial\overline{x}^O}\frac{\partial\overline{H}^C}{\partial R}}{\frac{\partial\overline{G}^C}{\partial\overline{x}^C}\frac{\partial\overline{H}^C}{\partial\overline{x}^O} - \frac{\partial\overline{G}^C}{\partial\overline{x}^O}\frac{\partial\overline{H}^C}{\partial\overline{x}^C}} \tag{A.2}$$

and

$$\frac{d\overline{x}^O}{dR} = \frac{-\frac{\partial\overline{G}^C}{\partial\overline{x}^C}\frac{\partial\overline{H}^C}{\partial R} + \frac{\partial\overline{G}^C}{\partial R}\frac{\partial\overline{H}^C}{\partial\overline{x}^C}}{\frac{\partial\overline{G}^C}{\partial\overline{x}^C}\frac{\partial\overline{H}^C}{\partial\overline{x}^O} - \frac{\partial\overline{G}^C}{\partial\overline{x}^O}\frac{\partial\overline{H}^C}{\partial\overline{x}^C}}. \tag{A.3}$$

Following Reinganum (1985), we assume that the denominators of both expressions can be interpreted as a stability condition and, hence, are positive.[24]

The numerator of (A.2) is equal to

$$-h\left(\overline{x}^O\right)\left(h'\left(\overline{x}^C\right)\frac{L}{r} - 1\right)\left[\begin{array}{c} h''\left(\overline{x}^O\right)\left[L + \overline{x}^O + \left(\left(R-C-1\right)h\left(\overline{x}^O\right) + Ch\left(\overline{x}^C\right)\right)\frac{L}{r}\right] \\ -h'\left(\overline{x}^O\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r} - 1\right)\end{array}\right] > 0.$$

The numerator of (A.3) is equal to

$$h\left(\overline{x}^O\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r} - 1\right)\left[\begin{array}{c} -h''\left(\overline{x}^C\right)\left(L + C\overline{x}^C + \left(R-C\right)h\left(\overline{x}^O\right)\frac{L}{r}\right) \\ +Ch'\left(\overline{x}^C\right)\left(h'\left(\overline{x}^C\right)\frac{L}{r} - 1\right)\end{array}\right] > 0.$$

Hence, we have $\frac{d\overline{x}^C}{dR}$ and $\frac{d\overline{x}^O}{dR} > 0$.

[24] See p. 92 in Reinganum (1985).

2 Proof of Proposition 1

The first step is to show that $x^C < x^O$. To do this, we evaluate the first order condition of the R&D cartel and of a single outside firm when all firms invest the same per firm amount x. This yields

$$h'(x)\left[L + Cx + (R - C)\,h(x)\,\frac{L}{r}\right] - [r + Rh(x)]$$

and

$$h'(x)\left[L + x + (R - 1)\,h(x)\,\frac{L}{r}\right] - [r + Rh(x)]$$

respectively. Subtracting the latter from the former gives

$$-(C-1)\,h'(x)\left[h(x)\,\frac{L}{r} - x\right] < 0.$$

This means that if it is optimal for the outsiders to invest x when the R&D cartel firms also invest x, the R&D cartel's best response to x is to invest less than x. Hence, it must be that $x^C < x^O$ in equilibrium.

We now show that outsiders must invest x^*, and therefore that if there are any active outsiders, the R&D cartel invests $x^C < x^*$. All active outsider firms in the R&D race must earn zero profits in equilibrium. Hence, by Lemma 1 such a firm i must face the same value of α_i as under R&D competition. Hence, this firm's maximization problem is the same as under R&D competition, and firm i will therefore invest x^* under R&D cartel too. Hence, from the above analysis, we have $x^C < x^*$ in equilibrium whenever there are outsiders active in the R&D race.

3 Proof of Lemma 3

$\overline{x}^J$ and $\overline{x}^O$ are defined by the following first order conditions:

$$\overline{G}^J \equiv Jh'\left(\overline{x}^J\right)\left[L^J + \overline{x}^J + \frac{L^J}{r}\,(R-J)\,h\left(\overline{x}^O\right)\right] - \left[r + Jh\left(\overline{x}^J\right) + (R-J)\,h\left(\overline{x}^O\right)\right] = 0 \tag{A.4}$$

and

$$\overline{H}^J \equiv h'\left(\overline{x}^O\right)\left[L+\overline{x}^O+\frac{L}{r}\left[(R-C-1)\,h\left(\overline{x}^O\right)+Jh\left(\overline{x}^J\right)\right]\right]-\left[r+Jh\left(\overline{x}^J\right)+(R-J)\,h\left(\overline{x}^O\right)\right]=0. \tag{A.5}$$

Totally differentiating these and applying Cramer's Rule gives

$$\frac{d\overline{x}^J}{dR}=\frac{-\frac{\partial\overline{G}^J}{\partial R}\frac{\partial\overline{H}^J}{\partial\overline{x}^O}+\frac{\partial\overline{G}^J}{\partial\overline{x}^O}\frac{\partial\overline{H}^J}{\partial R}}{\frac{\partial\overline{G}^J}{\partial\overline{x}^J}\frac{\partial\overline{H}^J}{\partial\overline{x}^O}-\frac{\partial\overline{G}^J}{\partial\overline{x}^O}\frac{\partial\overline{H}^J}{\partial\overline{x}^J}} \tag{A.6a}$$

and

$$\frac{d\overline{x}^O}{dR}=\frac{-\frac{\partial\overline{G}^J}{\partial\overline{x}^J}\frac{\partial\overline{H}^J}{\partial R}+\frac{\partial\overline{G}^J}{\partial R}\frac{\partial\overline{H}^J}{\partial\overline{x}^J}}{\frac{\partial\overline{G}^J}{\partial\overline{x}^J}\frac{\partial\overline{H}^J}{\partial\overline{x}^O}-\frac{\partial\overline{G}^J}{\partial\overline{x}^O}\frac{\partial\overline{H}^J}{\partial\overline{x}^J}}. \tag{A.7}$$

Following Reinganum (1985), we assume that the denominators of both expressions can be interpreted as a stability condition and, hence, are positive.[25]

The numerator of (A.6a) is equal to

$$-h\left(\overline{x}^O\right)\left(Jh'\left(\overline{x}^J\right)\frac{L^J}{r}-1\right)\left[\begin{array}{c} h''\left(\overline{x}^O\right)\left[L+\overline{x}^O+\frac{L}{r}\left((R-J-1)\,h\left(\overline{x}^O\right)+Ch\left(\overline{x}^J\right)\right)\right] \\ -h'\left(\overline{x}^O\right)\left(\frac{L}{r}h'\left(\overline{x}^O\right)-1\right)\end{array}\right]>0.$$

The numerator of (A.7) is equal to

$$Jh\left(\overline{x}^O\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r}-1\right)\left[\begin{array}{c} -h''\left(\overline{x}^J\right)\left[L^J+\overline{x}^J+\frac{L^J}{r}(R-J)\,h\left(\overline{x}^O\right)\right] \\ +h'\left(\overline{x}^J\right)\left(Jh'\left(\overline{x}^J\right)\frac{L^J}{r}-1\right)\end{array}\right]>0.$$

Hence, we have $\frac{d\overline{x}^J}{dR}$ and $\frac{d\overline{x}^J}{dR}>0$.

4 Proof of Lemma 4

The free entry equilibrium investment levels and the number of firms are implicitly defined by the following three equations.

$$G^J \equiv Jh'\left(x^J\right)\left[L^J+x^J+\frac{L^J}{r}\left(R^J-J\right)h\left(x^O\right)\right]-\left[r+Jh\left(x^J\right)+\left(R^J-J\right)h\left(x^O\right)\right]=0, \tag{A.8}$$

$$H^J \equiv h'\left(x^O\right)\left[L+x^O+\frac{L}{r}\left[\left(R^J-J-1\right)h\left(x^O\right)+Jh\left(x^J\right)\right]\right]-\left[r+Jh\left(x^J\right)+\left(R^J-J\right)h\left(x^O\right)\right]=0 \tag{A.9}$$

[25] See p. 92 in Reinganum (1985).

and

$$Z^J \equiv \frac{h\left(x^O\right)\frac{L}{r} - x^O}{r + Jh\left(x^J\right) + \left(R^J - J\right)x^O} - S = 0. \tag{A.10}$$

Totally differentiating and applying Cramer's Rule gives us

$$\frac{dx^J}{dL^J} = \frac{\begin{vmatrix} -\frac{\partial G^J}{\partial L^J} & \frac{\partial G^J}{\partial x^O} & \frac{\partial G^J}{\partial R} \\ -\frac{\partial H^J}{\partial L^J} & \frac{\partial H^J}{\partial x^O} & \frac{\partial H^J}{\partial R} \\ -\frac{\partial Z^J}{\partial L^J} & \frac{\partial Z^J}{\partial x^O} & \frac{\partial Z^J}{\partial R} \end{vmatrix}}{\begin{vmatrix} \frac{\partial G^J}{\partial x^J} & \frac{\partial G^J}{\partial x^O} & \frac{\partial G^J}{\partial R} \\ \frac{\partial H^J}{\partial x^J} & \frac{\partial H^J}{\partial x^O} & \frac{\partial H^J}{\partial R} \\ \frac{\partial Z^J}{\partial x^J} & \frac{\partial Z^J}{\partial x^O} & \frac{\partial Z^J}{\partial R} \end{vmatrix}}$$

$$= \frac{\frac{\partial G^J}{\partial L^J}\left[\frac{\partial Z^J}{\partial x^O}\frac{\partial H^J}{\partial R} - \frac{\partial Z^J}{\partial R}\frac{\partial H^J}{\partial x^O}\right]}{\frac{\partial Z^J}{\partial x^J}\left[\frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial R} - \frac{\partial G^J}{\partial R}\frac{\partial H^J}{\partial x^O}\right] - \frac{\partial Z^J}{\partial x^O}\left[\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial R} - \frac{\partial G^J}{\partial R}\frac{\partial H^J}{\partial x^J}\right] + \frac{\partial Z^J}{\partial R}\left[\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial x^O} - \frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial x^J}\right]}.$$

The stability condition implies that $\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial x^O} > \frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial x^J}$. Since we have $\frac{\partial G^J}{\partial x^J} < 0$ while both $\frac{\partial G^J}{\partial x^O}$ and $\frac{\partial H^J}{\partial x^J}$ are > 0, this means we must have $\frac{\partial H^J}{\partial x^O} < 0$. Furthermore,

$$sign\left\{\frac{\partial G^J}{\partial L^J}\right\} = sign\left\{\frac{\partial H^J}{\partial R}\right\} = sign\left\{\frac{\partial G^J}{\partial R}\right\} = sign\left\{\frac{\partial G^J}{\partial x^O}\right\} = sign\left\{\frac{\partial H^J}{\partial x^J}\right\}$$

which is positive, and

$$sign\left\{\frac{\partial G^J}{\partial x^J}\right\} = sign\left\{\frac{\partial H^J}{\partial x^O}\right\} = sign\left\{\frac{\partial Z^J}{\partial R}\right\} = sign\left\{\frac{\partial Z^J}{\partial x^O}\right\} = sign\left\{\frac{\partial Z^J}{\partial x^J}\right\}$$

which is negative. These imply that both the numerator and denominator of $\frac{\partial x^J}{\partial L^J}$ are negative and, hence, we have $\frac{dx^J}{dL^J} > 0$.

To prove that equilibrium RJV cartel profits are monotonically increasing in L^J, note that

$$\frac{d\Pi^J}{dL^J} = \frac{\partial \Pi^J}{\partial L^J} + \frac{\partial \Pi^J}{\partial \alpha^J}\frac{\partial \alpha^J}{\partial L^J}$$

The first term on the right hand side is positive by inspection. The first part of the second term is negative by inspection, and the second part is negative because as Lemma **??** shows, increasing L^J decreases the number of outside firms without changing their per-firm investment.

5 Proof of Lemma 5

Substituting for $L^J = \frac{L}{J}$ in the first derivative of (9) with respect to x_i reveals that if $C = J$, i.e., if an R&D cartel and an RJV cartel both have the same number of firms, the per-firm investment level is the same under both types of cooperative arrangements. Similarly, substituting for $L^J = \frac{L}{J}$ and the equilibrium payoff level in (11) shows that the profits are also the same under the two cooperative arrangements. Hence, the results follow from Lemma 1 and Proposition 3.

For the case of $L^J = L$, we first evaluate the first order conditions of the RJV cartel and of an outsider firm when all firms invest the same per firm amount, x. These first order conditions then will be

$$Jh'(x)\left[L + x + \frac{L}{r}(R - J - 1)h(x) + \frac{L}{r}h(x)\right] - [r + Rh(x)]$$

and

$$h'(x)\left[L + x + \frac{L}{r}(R - J - 1)h(x) + J\frac{L}{r}h(x)\right] - [r + Rh(x)] = 0$$

respectively. Subtracting the latter from the former gives

$$(J-1)h'(x)\left[L + x + \frac{L}{r}(R - J - 1)h(x)\right] > 0.$$

This means that if it is optimal for the outsiders to invest x when the RJV cartel firms also invest x, the RJV cartel's best response to x is to invest more than x. Hence, it must be that $x^J > x^O$ in equilibrium.

We now show that any active outsiders must invest x^*, and therefore that if there are any active outsiders, the RJV cartel invests $x^J > x^*$. All active outsider firms in the R&D race must earn zero profits in equilibrium. Hence, by Lemma 1 such a firm i must face the same value of α_i as under R&D competition. Hence, this firm's maximization problem is the same as under R&D competition, and firm i will therefore invest x^* under RJV cartel too. Hence, from the above analysis, we have $x^J > x^*$ in equilibrium.

To see that the RJV cartel earns positive profits, note that if we hold the outsiders' investments constant at x^* and decrease the RJV cartel's investment to x^*, the RJV cartel's per firm profits are

$$\frac{\Pi^J}{J} = \pi^J = \frac{h(x^*)\frac{JL}{r} - x^*}{r + Jh(x^*) + (R^J - J)h(x^*)} - S$$

and an outside firm earns

$$\Pi_i = \frac{h(x^*)\frac{L}{r} - x^*}{r + Jh(x^*) + (R^J - J)h(x^*)} - S$$

which is clearly less. However, the outside firm would be earning strictly positive profits, since there would be fewer firms in total making the same per-firm investments as under R&D competition. Hence, the RJV cartel would also be making strictly positive profits. Since the RJV cartel chooses x^J to maximize its joint profits given the outside firms choose x^*, it must earn higher profits still in equilibrium.

6 Proof of Proposition 4

We know from Lemma 5 that when $L^J = \frac{L}{J}$, the members of the RJV earn less than they would under R&D competition and when $L^J = L$, they earn more than they would under R&D competition. Hence, given Lemma 4, there must exist a critical value $\widehat{L}^J \in \left(\frac{L}{J}, L\right)$ above which the profits with an RJV cartel of size J are higher than they are under R&D competition and below which they are lower.

Similarly, we know from Lemma 5 that when $L^J = \frac{L}{J}$, the members of the RJV invest less than they would under R&D competition and when $L^J = L$, they invest more than they would under R&D competition. Hence, given Lemma 4, there must exist a critical value $\widetilde{L}^J \in \left(\frac{L}{J}, L\right)$ above which the per-firm investment level with an RJV cartel of size J is higher than it is under R&D competition and below which it is lower.

To prove that $\widetilde{L}^J > \widehat{L}^J$, we evaluate the profitability of an RJV cartel when $L^J = \widetilde{L}^J$ and show that it is positive. When $L^J = \widetilde{L}^J$, the RJV cartel's equilibrium per-firm investment is x^* by definition. Note that each outsider participant in the R&D race in equilibrium earns

$$\pi^O = \frac{h(x^*)\frac{L}{r} - x^*}{r + Jh(x^*) + (R^J - J)h(x^*)} - S = 0$$

while each member of the RJV cartel earns

$$\frac{\Pi^J}{J} = \pi^J = \frac{h(x^*)\frac{J\widetilde{L}^J}{r} - x^*}{r + Jh(x^*) + (R^J - J)h(x^*)} - S$$

Subtracting π^O from π^J yields

$$\frac{h(x^*)\left[\frac{J\widetilde{L}^J - L}{r}\right]}{r + Jh(x^*) + (R^J - J)h(x^*)} > 0 \tag{A.11}$$

since $\widetilde{L}^J > L$.

7 Cooperative R&D without R&D race outsiders

Proposition 7 *R&D cartels invest less per-firm than the per-firm investment level under R&D competition and are unprofitable.*

We first show that $x^C < x^*$ in equilibrium. If no outside firms are active in the R&D race in equilibrium, the first order condition of the R&D cartel reduces to

$$h'\left(x^C\right)\left[L + Cx^C\right] - \left[r + Ch\left(x^C\right)\right] = 0.$$

Using the implicit function theorem, we have

$$\frac{\partial x^C}{\partial C} = -\frac{h'\left(x^C\right)x^C - h\left(x^C\right)}{h''\left(x^C\right)\left[L + Cx^C\right]}.$$

The denominator is clearly negative because $h\left(x^C\right)$ is concave, and the numerator is equal to zero at $x^C = 0$ and is strictly decreasing in x^C for all $x^C > 0$. Hence, $\frac{\partial x^C}{\partial C} < 0$ whenever there are no active outsider firms. Note that when there are active outsider firms earning zero profits, the number of such firms is defined by the following equality:

$$R^* h(x^*) = Oh(x^*) + Ch\left(x^C\right) \tag{A.12}$$

where $R^* h(x^*)$ is the aggregate rate of innovation under R&D competition and O is the number of outsiders in equilibrium in the presence of the R&D cartel. Clearly it is the case that $\lim_{O \to 0} Ch\left(x^C\right) = R^* h(x^*)$. Since we have $x^C < x^O = x^*$ as $O \to 0$ from above, it must be that as $O \to 0$ we have $C > R^*$. Define the value of C for which $O = 0$ to be $\widehat{C}$. Then $x^C\left(\widehat{C}\right) < x^*$. Since we have $\frac{\partial x^C}{\partial C} < 0$ for all $C \geqslant \widehat{C}$, we can now conclude that $x^C < x^*$ when there are no active R&D race outsiders.

We next show that R&D cartels are unprofitable when there are no outsiders in the race. Define the value of C for which $Ch\left(x^C\right) = R^* h\left(x^*\right)$ to be $\widehat{C}$. When $C = \widehat{C}$ the per-firm profit of the R&D cartel members is negative. For all C such that there are no outsiders, we can use (9) to write the equilibrium per-firm R&D cartel payoff as

$$\pi^C = \frac{h\left(x^C\right)\frac{L}{r} - x^C}{r + Ch\left(x^C\right)} - S.$$

Applying the envelope theorem, we have $\frac{\partial \pi^C}{\partial C} < 0$. Hence, $C\pi^C < 0$ for all C such that there are no outsiders in equilibrium.

For the case of RJV cartels we have the following proposition.

Proposition 8 *Members of an RJV cartel invest higher (lower) amounts per-firm than under R&D competition for values of J such that $L^J > \widetilde{L}^J(J) \in \left(\frac{L}{J}, L\right)$ ($L^J < \widetilde{L}^J(J) \in \left(\frac{L}{J}, L\right)$). Members of an RJV cartel earn higher (lower) profits than under R&D competition for values of J such that $L^J > \widehat{L}^J(J) \in \left(\frac{L}{J}, L\right)$ ($L^J < \widehat{L}^J(J) \in \left(\frac{L}{J}, L\right)$).*

We first show that the per-firm investment and profits of an RJV cartel are monotonically increasing in L^J when there are no outsiders in the R&D race. In this case, we have

$$\frac{\partial x^J}{\partial L^J} = -\frac{Jh'\left(x^J\right)}{Jh''\left(x^J\right)\left[L^J + x^J\right]} > 0$$

and

$$\frac{d\Pi^J}{dL^J} > 0.$$

Now, when $L^J = \frac{L}{J}$, the first order condition and equilibrium profit function of an RJV cartel are identical to those of an R&D cartel of the same size. Hence, since from Proposition 7 we know that R&D cartels invest less than x^* per firm and are unprofitable, the same must be true of RJV cartels when $L^J = \frac{L}{J}$.

For the case where $L^J = L$, the first order condition of the RJV cartel reduces to

$$Jh'\left(x^J\right)\left[L + x^J\right] - \left[r + Jh\left(x^J\right)\right] = 0.$$

Using the implicit function theorem, we have

$$\frac{\partial x^J}{\partial J} = -\frac{h'\left(x^J\right)\left[L + x^J\right] - h\left(x^J\right)}{Jh''\left(x^J\right)\left[L + x^J\right]}.$$

The denominator is clearly negative, and the first order condition implies that the numerator is equal to $\frac{r}{J} > 0$, so we have $\frac{\partial x^J}{\partial J} > 0$ when there are no outsiders in the R&D race and we have fixed $L^J = L$. Holding L^J fixed at L, define $\widehat{J}$ as the number of RJV participants for which there are exactly zero outsiders in the R&D race. As J approaches $\widehat{J}$ we have $x^J > x^*$, and since $\frac{\partial x^J}{\partial J} > 0$ for all J such that there are no outsiders, we must also have $x^J > x^*$ when there are no outsiders active in the R&D race.

To see that profits are positive, note that at $\widehat{J}$ we have strictly positive profits. When there are no outsiders in the R&D race, we can re-write the RJV cartel's per firm payoff as

$$\frac{\Pi^J}{J} = \pi^J = \frac{h\left(x^J\right)\frac{JL}{r} - x^J}{r + Jh\left(x^J\right)} - S.$$

Applying the envelope theorem gives

$$\frac{\partial \pi^J}{\partial J} = \frac{rh\left(x^J\right)\frac{L}{r} + h\left(x^J\right)x^J}{\left[r + Jh\left(x^J\right)\right]^2} > 0.$$

The proposition follows then from the fact that per-firm investment and profits are increasing in L^J, and are less than under R&D competition when $L^J = \frac{L}{J}$ but are greater than under R&D competition when $L^J = L$.